DIMACS Series in Discrete Mathematics
and Theoretical Computer Science

A list of corrections is attached to the end.

# Catalog of All Fullerenes with Ten or More Symmetries

Jack E. Graver

By a fullerene, we mean a trivalent plane graph $\Gamma = (V, E, F)$ with only hexagonal and pentagonal faces. It follows easily from Euler's Formula that each fullerene has exactly 12 pentagonal faces. The simplest fullerene is the graph of the dodecahedron with 12 pentagonal faces and no hexagonal faces. It is frequently easier to work with the duals to the fullerenes: geodesic domes, i.e. triangulations of the sphere with vertices of degree 5 and 6. It is in this context that Goldberg[**5**], Caspar and Klug[**1**] and Coxeter[**2**] parameterized the geodesic domes/fullerenes that include the full rotational group of the icosahedron among their symmetries. In this catalog we extend their work by giving a complete parameterization of all fullerenes with ten or more symmetries. In their model, the faces of the icosahedron are filled in with equilateral triangles from the hexagonal tessellation of the plane. In [**6**], the author generalized their method to other plane graphs filling in the faces with other polygonal regions from the hexagonal tessellation of the plane. Assigned to each fullerene is a 12-vertex plane graphs with edge and angle labels called its signatures. The fullerene can then be reconstituted from its signature by gluing together the regions from the hexagonal tessellation of the plane corresponding to the faces of its signature. To distinguish between edges in the graph model of a fullerene and the edges of its signature, the latter are referred to as segments. The region corresponding to a face is completely determined by the "Coxeter coordinates" of its segments and the "types" of the angles between segments. In Figure 1, we have drawn several segments. Referring to that figure the left hand segments have Coxeter coordinates $(4, 2)$ and $(1, 5)$, respectively, and subtend an angle of type 2. The thinner lines show how the Coxeter coordinates are determined. The type of an angle is the number of centers of edges of the central hexagon between the segments. Segments which run through successive centers are assigned a single Coxeter coordinate and contribute $\frac{1}{2}$ to each of the angle types on either side. This is illustrated by the isosceles triangle at the right in the figure.

One may think of the segments of a signature as the shortest segments joining the centers of pentagonal faces in a spherical representation of a fullerene. Since only the shortest segments necessary to connect all pentagons are included, there are tight restrictions on the polygons in the hexagonal tessellation that can be faces of a signature. For example there are only six types of triangles that can occur. These are pictured in the next figure. We present these triangles with variable Coxeter coordinates. The variables are independent positive integers; any assignment

2000 *Mathematics Subject Classification.* 05C05, 92E10.
*Key words and phrases.* Fullerenes, geodesic domes.



©2005 American Mathematical Society

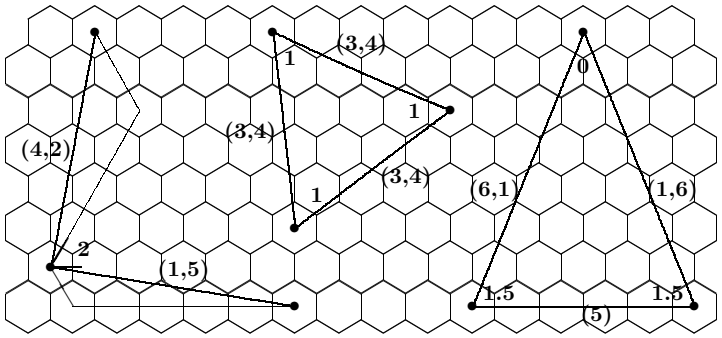



FIGURE 1.

of values to these variables will give a specific triangle. Since we are interested in symmetry, it is useful to consider the symmetries of these triangles. First we note all symmetries preserve angle types and that an orientation preserving symmetry must map each segment onto a segment with the same Coxeter coordinates while orientation reversing symmetry maps each segment onto a segment with reversed Coxeter coordinates. Thus the first two triangles in Figure 2 admit only the reflection through the vertical axis; the second two admit only rotations and each is reflected into the other; the last two admit all six of the symmetries of an equilateral triangle.

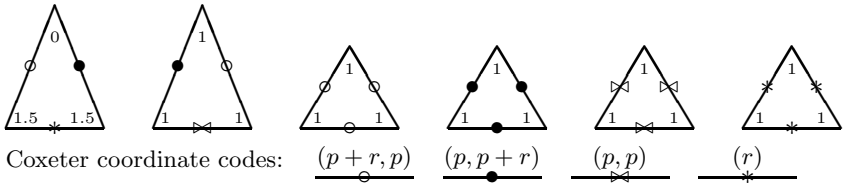
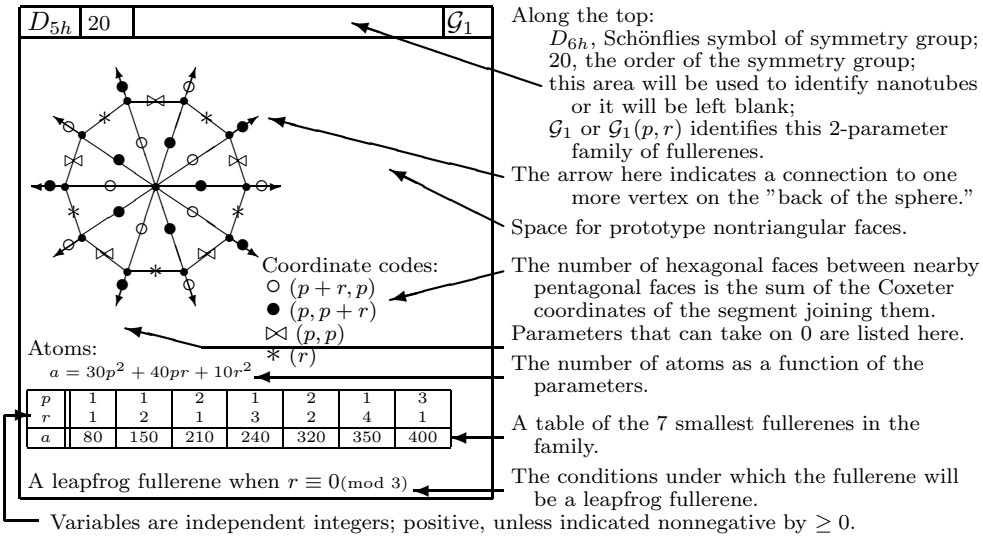



FIGURE 2.

This catalog has a complete listing of all fullerenes with symmetry groups $I_h$, $I$, $T$, $D_{6h}$, $D_{6d}$, $D_6$, $D_{5h}$, $D_{5d}$, $D_5$, $D_{3h}$ or $D_{3d}$. They fall into 112 infinite families; each family is represented by a single entry in the catalog. Below is a typical catalog entry and an explanation of its features. If a fullerene (signature) does not admit a reflection then only one of that fullerene (signature) and its mirror image is included in the catalog. The case by case considerations that lead to this catalog are given in the accompanying paper [**7**]. There it is proved that every fullerene with ten or more symmetries is listed once and only once in this catalog. Fowler and Manolopoulos [**4**] showed that there were only 28 fullerene symmetry groups 11 of which are explored here. In the following table we list the groups, using the Schönflies symbols, the group orders, the number of classes with that group and the page where the listings for that group start.

| *Group* | *order* | # | *page* | *Group* | *order* | # | *page* | *Group* | *order* | # | *page* |
|---|---|---|---|---|---|---|---|---|---|---|---|
| $I_h$ | 120 | 2 | 4 | | | | | $I$ | 60 | 1 | 4 |
| $T_h$ | 24 | 3 | 4 | $T_d$ | 24 | 3 | 5 | $T$ | 12 | 15 | 5 |
| $D_{6h}$ | 24 | 3 | 8 | $D_{6d}$ | 24 | 7 | 8 | $D_6$ | 12 | 17 | 9 |
| $D_{5h}$ | 20 | 3 | 12 | $D_{5d}$ | 20 | 5 | 13 | $D_5$ | 10 | 17 | 13 |
| $D_{3h}$ | 12 | 18 | 16 | $D_{3d}$ | 12 | 18 | 19 | | | | |

**A typical catalog entry.**




In Figure 3, we have drawn the fullerene in this family given by the parameter values $p = 1$, $r = 2$, $\mathcal{G}_1(1,2)$. Eleven of the pentagonal faces are shaded, the twelfth is the outside face. The segments forming the "equator" of this fullerene are indicated by the heavy circle and heavy lines demark the remaining segments bounding two of the triangular faces of the signature, one of each type.

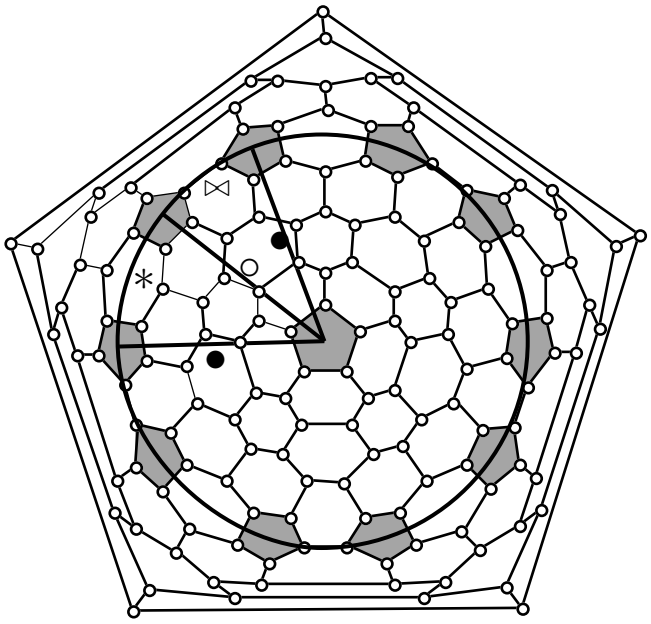
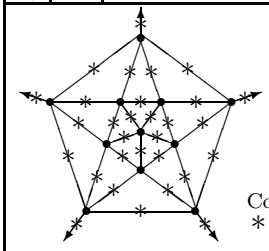


FIGURE 3.

## $I_h$ | 120 | $\mathcal{A}_1$



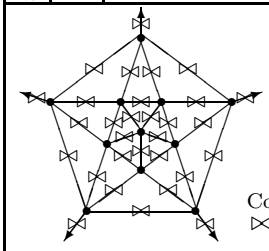
Coordinate codes:
$*$ $(r)$

Atoms:
$a = 20r^2$

| $r$ | 1 | 2 | 3 | 4 | 5 | 6 | 7 |
|---|---|---|---|---|---|---|---|
| $a$ | 20 | 80 | 180 | 320 | 500 | 720 | 980 |

A leapfrog fullerene when $r \equiv 0 \pmod 3$

## $I_h$ | 120 | $\mathcal{A}_2$



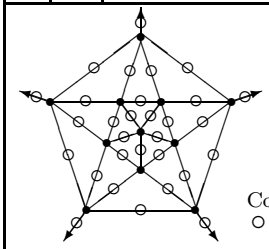
Coordinate codes:
$\bowtie$ $(p, p)$

Atoms:
$a = 60p^2$

| $p$ | 1 | 2 | 3 | 4 | 5 | 6 | 7 |
|---|---|---|---|---|---|---|---|
| $a$ | 60 | 240 | 540 | 960 | 1500 | 2160 | 2946 |

All are leapfrog fullerenes

## $I$ | 60 | $\mathcal{A}_3$



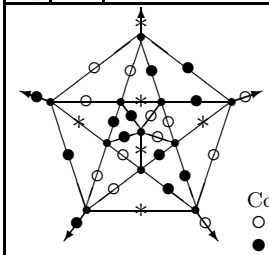
Coordinate codes:
$\circ$ $(p + r, p)$

Atoms:
$a = 60p^2 + 60pr + 20r^2$

| $p$ | 1 | 1 | 2 | 1 | 2 | 1 | 3 |
|---|---|---|---|---|---|---|---|
| $r$ | 1 | 2 | 1 | 3 | 2 | 4 | 1 |
| $a$ | 140 | 260 | 380 | 420 | 560 | 620 | 740 |

A leapfrog fullerene when $r \equiv 0 \pmod 3$

## $T_h$ | 24 | $\mathcal{A}_8$



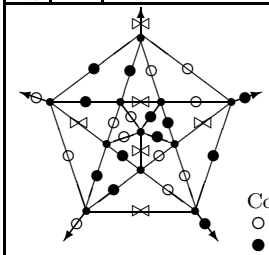
Coordinate codes:
$\circ$ $(p + r, p)$
$\bullet$ $(p, p + r)$
$*$ $(r)$

Atoms:
$a = 24p^2 + 48pr + 20r^2$

| $p$ | 1 | 1 | 2 | 1 | 2 | 3 | 1 |
|---|---|---|---|---|---|---|---|
| $r$ | 1 | 2 | 1 | 3 | 2 | 1 | 4 |
| $a$ | 92 | 200 | 212 | 348 | 368 | 380 | 536 |

A leapfrog fullerene when $r \equiv 0 \pmod 3$

## $T_h$ | 24 | $\mathcal{A}_9$



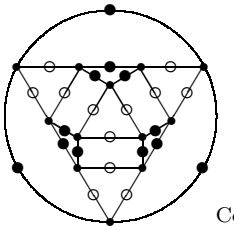
Coordinate codes:
$\circ$ $(p + r, p)$
$\bullet$ $(p, p + r)$
$\bowtie$ $(p, p)$

Atoms:
$a = 60p^2 + 48pr + 8r^2$

| $p$ | 1 | 1 | 1 | 2 | 1 | 2 | 1 |
|---|---|---|---|---|---|---|---|
| $r$ | 1 | 2 | 3 | 1 | 4 | 2 | 5 |
| $a$ | 116 | 188 | 276 | 344 | 380 | 464 | 500 |

A leapfrog fullerene when $r \equiv 0 \pmod 3$

## $T_h$ | 24 | $\mathcal{B}_1$



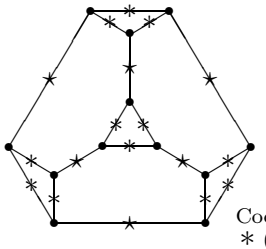

Coordinate codes:
$\bullet$ $(p, p + r)$
$\circ$ $(p + r, p)$

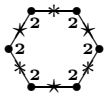
Atoms:
$a = 60p^2 + 72pr + 20r^2$

| $p$ | 1 | 1 | 2 | 1 | 2 | 1 | 3 |
|---|---|---|---|---|---|---|---|
| $r$ | 1 | 2 | 1 | 3 | 2 | 4 | 1 |
| $a$ | 152 | 284 | 404 | 456 | 608 | 668 | 776 |

A leapfrog fullerene when $r \equiv 0 \pmod 3$

## $T_d$ | 24 | $\mathcal{C}_1$





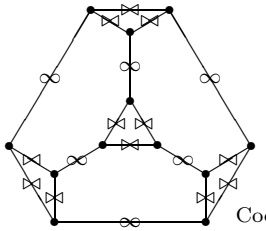

Coordinate codes:
$*$ $(r)$
$\star$ $(s)$

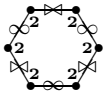
Atoms:
$a = 8r^2 + 16rs + 4s^2$

| $r$ | 1 | 1 | 2 | 1 | 2 | 3 | 1 |
|---|---|---|---|---|---|---|---|
| $s$ | 1 | 2 | 1 | 3 | 2 | 1 | 4 |
| $a$ | 28 | 56 | 68 | 92 | 112 | 124 | 136 |

A leapfrog fullerene when $r \equiv s \equiv 0 \pmod 3$

## $T_d$ | 24 | $\mathcal{C}_2$





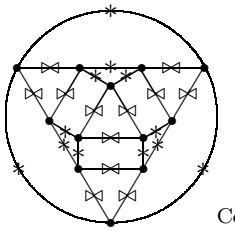

Coordinate codes:
$\bowtie$ $(p, p)$
$\infty$ $(q, q)$

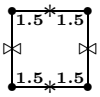
Atoms:
$a = 24p^2 + 48pq + 12q^2$

| $p$ | 1 | 1 | 2 | 1 | 2 | 3 | 1 |
|---|---|---|---|---|---|---|---|
| $q$ | 1 | 2 | 1 | 3 | 2 | 1 | 4 |
| $a$ | 84 | 168 | 204 | 276 | 336 | 372 | 408 |

All are leapfrog fullerenes

## $T_d$ | 24 | $\mathcal{B}_2$





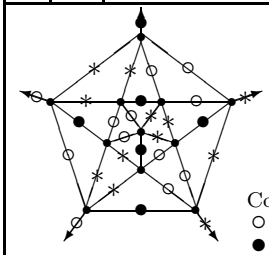

Coordinate codes:
$\bowtie$ $(p, p)$
$*$ $(r)$

Atoms:
$a = 12p^2 + 24pr + 4r^2$

| $p$ | 1 | 1 | 2 | 1 | 2 | 1 | 3 |
|---|---|---|---|---|---|---|---|
| $r$ | 1 | 2 | 1 | 3 | 2 | 4 | 1 |
| $a$ | 40 | 76 | 100 | 120 | 160 | 172 | 184 |

A leapfrog fullerene when $r \equiv 0 \pmod 3$

## $T$ | 12 | $\mathcal{A}_{10}$



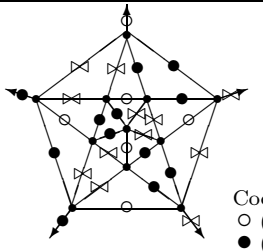
Coordinate codes:
$\circ$ $(p + r, p)$
$\bullet$ $(p, p + r)$
$*$ $(r)$

Atoms:
$a = 12p^2 + 36pr + 20r^2$

| $p$ | 1 | 2 | 1 | 3 | 2 | 1 | 4 |
|---|---|---|---|---|---|---|---|
| $r$ | 1 | 1 | 2 | 1 | 2 | 3 | 1 |
| $a$ | 68 | 140 | 164 | 236 | 272 | 300 | 356 |

A leapfrog fullerene when $r \equiv 0 \pmod 3$

## $T$ | 120 | $\mathcal{A}_{11}$



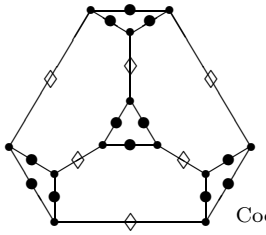
Coordinate codes:
$\circ$ $(p + r, p)$
$\bullet$ $(p, p + r)$
$\bowtie$ $(p, p)$

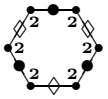
Atoms:
$a = 60p^2 + 36pr + 4r^2$

| $p$ | 1 | 1 | 1 | 1 | 2 | 1 | 2 |
|---|---|---|---|---|---|---|---|
| $r$ | 1 | 2 | 3 | 4 | 1 | 5 | 2 |
| $a$ | 100 | 148 | 204 | 268 | 316 | 340 | 400 |

A leapfrog fullerene when $r \equiv 0 \pmod 3$

## $T$ | 12 | $\mathcal{C}_3$





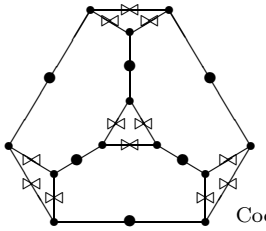

Coordinate codes:
$\bullet$ $(p, p + q)$
$\diamond$ $(r, s)$

Atoms:
$a = 24p^2 + 8q^2 + 4r^2 + 4s^2 +$
$24pq + 24pr + 24ps + 8qr + 16qs + 4rs$

| $p$ | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
|---|---|---|---|---|---|---|---|
| $q$ | 1 | 1 | 1 | 2 | 1 | 1 | 1 |
| $r$ | 1 | 2 | 1 | 1 | 3 | 2 | 1 |
| $s$ | 1 | 1 | 2 | 1 | 1 | 2 | 3 |
| $a$ | 140 | 188 | 196 | 212 | 244 | 248 | 260 |

A leapfrog fullerene when $q \equiv 0$ & $r \equiv s \pmod 3$

$T$ | 12 | $\mathcal{C}_4$



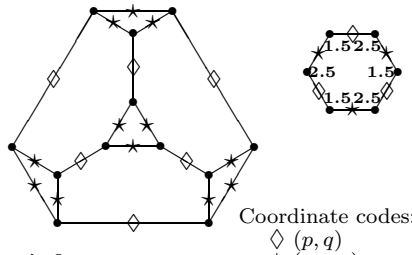

Coordinate codes:
⋈ $(p, p)$
● $(r, r+s)$

Atoms:
$a = 24p^2 + 12r^2 + 4s^2 + 48pr + 24ps + 12rs$

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| $p$ | 1 | 1 | 1 | 1 | 2 | 1 | 1 |
| $r$ | 1 | 1 | 2 | 1 | 1 | 2 | 1 |
| $s$ | 1 | 2 | 1 | 3 | 1 | 2 | 4 |
| $a$ | 124 | 172 | 220 | 228 | 268 | 280 | 292 |

A leapfrog fullerene when $s \equiv 0 \pmod 3$

$T$ | 12 | $\mathcal{C}_5$




Coordinate codes:
◊ $(p, q)$
★ $(p+r)$

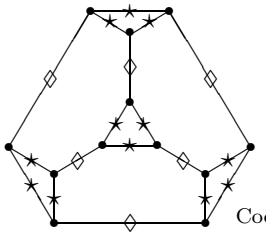
$r \geq 0$

Atoms:
$a = 20p^2 + 4q^2 + 8r^2 + 20pq + 24pr + 16qr$

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| $p$ | 1 | 1 | 1 | 1 | 2 | 1 | 1 |
| $q$ | 1 | 2 | 1 | 3 | 1 | 2 | 1 |
| $r$ | 0 | 0 | 1 | 0 | 0 | 1 | 2 |
| $a$ | 44 | 76 | 92 | 116 | 124 | 140 | 156 |

A leapfrog fullerene when $p \equiv q \equiv -r \pmod 3$

$T$ | 12 | $\mathcal{C}_6$



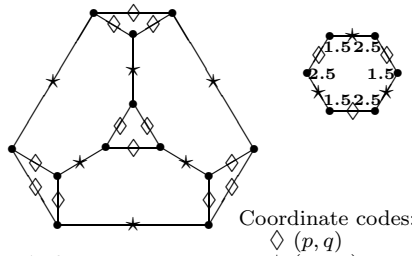

Coordinate codes:
◊ $(p+r, p+s)$
★ $(r)$

Atoms:
$a = 12p^2 + 20r^2 + 4s^2 + 36pr + 12ps + 20rs$

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| $p$ | 1 | 1 | 2 | 1 | 1 | 2 | 1 |
| $r$ | 1 | 1 | 1 | 1 | 2 | 1 | 1 |
| $s$ | 1 | 2 | 1 | 3 | 1 | 2 | 4 |
| $a$ | 104 | 148 | 188 | 200 | 220 | 244 | 260 |

A leapfrog fullerene when $r \equiv s \equiv 0 \pmod 3$

$T$ | 12 | $\mathcal{C}_7$




Coordinate codes:
◊ $(p, q)$
★ $(q+r)$

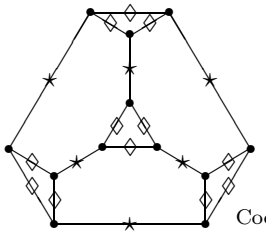
$r \geq 0$

Atoms:
$a = 8p^2 + 20q^2 + 4r^2 + 24pq + 16pr + 16qr$

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| $p$ | 1 | 1 | 2 | 1 | 1 | 2 | 3 |
| $q$ | 1 | 1 | 1 | 1 | 2 | 1 | 1 |
| $r$ | 0 | 1 | 0 | 2 | 0 | 1 | 0 |
| $a$ | 52 | 88 | 100 | 132 | 136 | 152 | 164 |

Leapfrog fullerene when $p \equiv q \equiv -r \pmod 3$

$T$ | 12 | $\mathcal{C}_8$



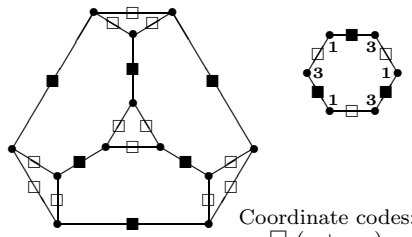

Coordinate codes:
◊ $(p+s, p+r)$
★ $(r)$

Atoms:
$a = 24p^2 + 20r^2 + 8s^2 + 48pr + 24ps + 24rs$

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| $p$ | 1 | 1 | 1 | 2 | 1 | 1 | 2 |
| $r$ | 1 | 1 | 2 | 1 | 1 | 2 | 1 |
| $s$ | 1 | 2 | 1 | 1 | 3 | 2 | 2 |
| $a$ | 148 | 220 | 280 | 292 | 308 | 376 | 388 |

Leapfrog fullerene when $r \equiv s \equiv 0 \pmod 3$

$T$ | 12 | $\mathcal{C}_9$



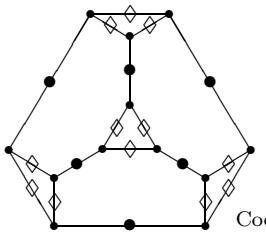

Coordinate codes:
□ $(p+r, q)$
■ $(p, q+s)$

Atoms:
$a = 4(5p^2 + 5q^2 + 2r^2 + s^2 + 5pq + 6pr + 5ps + 6qr + 4qs + 4rs)$

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| $p$ | 1 | 1 | 1 | 1 | 2 | 1 | 1 |
| $q$ | 1 | 1 | 1 | 2 | 1 | 1 | 1 |
| $r$ | 1 | 1 | 2 | 1 | 1 | 1 | 2 |
| $s$ | 1 | 2 | 1 | 1 | 1 | 3 | 2 |
| $a$ | 172 | 236 | 260 | 292 | 296 | 308 | 340 |

Leapfrog fullerene when $p - q \equiv -r \equiv s \pmod 3$

$T$ | 12 | $\mathcal{C}_{10}$



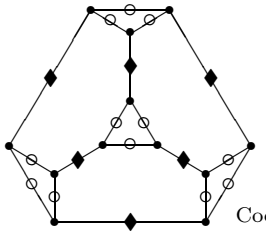

Coordinate codes:
◇ $(p+r, p+s)$
● $(p, p+s)$

Atoms:
$a = 4(15p^2 + 5s^2 + 2r^2 + 12pr + 15ps + 6rs)$

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| $p$ | 1 | 1 | 1 | 1 | 1 | 2 | 1 |
| $r$ | 1 | 2 | 1 | 3 | 2 | 1 | 1 |
| $s$ | 1 | 1 | 2 | 1 | 2 | 1 | 3 |
| $a$ | 220 | 316 | 364 | 428 | 484 | 508 | 548 |

A leapfrog fullerene when $r \equiv s \equiv 3 \pmod 3$

$T$ | 12 | $\mathcal{C}_{11}$



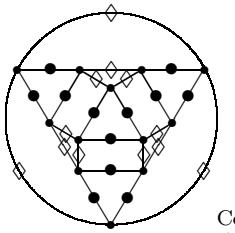

Coordinate codes:
○ $(p+s, p)$
⧫ $(p+s, p+r)$

Atoms:
$a = 4(15p^2 + 5s^2 + r^2 + 9pr + 15ps + 5rs)$

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| $p$ | 1 | 1 | 1 | 1 | 1 | 1 | 2 |
| $r$ | 1 | 2 | 1 | 3 | 2 | 4 | 1 |
| $s$ | 1 | 1 | 2 | 1 | 2 | 1 | 1 |
| $a$ | 200 | 268 | 340 | 344 | 428 | 428 | 476 |

A leapfrog fullerene when $r \equiv s \equiv 0 \pmod 3$

$T$ | 12 | $\mathcal{B}_3$



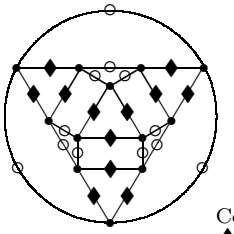

Coordinate codes:
● $(p, p+q+s)$
◇ $(p+r, p+s)$

Atoms:
$a = 4(15p^2 + q^2 + r^2 + 5s^2 +$
$9pq + 9pr + 15ps + 3qr + 5qs + 4rs)$

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| $p$ | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| $q$ | 1 | 1 | 2 | 1 | 1 | 3 | 2 |
| $r$ | 1 | 2 | 1 | 1 | 3 | 1 | 2 |
| $s$ | 1 | 1 | 1 | 2 | 1 | 1 | 1 |
| $a$ | 268 | 344 | 348 | 424 | 428 | 436 | 436 |

A leapfrog fullerene when $p \equiv q$ & $r \equiv s \pmod 3$

$T$ | 12 | $\mathcal{B}_4$



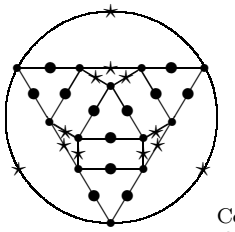

Coordinate codes:
⧫ $(p, p+r+s)$
○ $(p+r, p)$

Atoms:
$a = 4(15p^2 + 5r^2 + s^2 + 18pr + 9ps + 5rs)$

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| $p$ | 1 | 1 | 1 | 1 | 1 | 1 | 2 |
| $r$ | 1 | 1 | 1 | 2 | 1 | 2 | 1 |
| $s$ | 1 | 2 | 3 | 1 | 4 | 2 | 1 |
| $a$ | 212 | 280 | 356 | 364 | 440 | 452 | 500 |

A leapfrog fullerene when $p \equiv q$ & $r \equiv s \pmod 3$

$T$ | 12 | $\mathcal{B}_5$



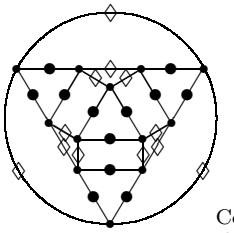

Coordinate codes:
● $(p, p+r)$
★ $(r+s)$

Atoms:
$a = 4(3p^2 + 5r^2 + s^2 + 9pr + 6ps + 5rs)$

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| $p$ | 1 | 1 | 2 | 1 | 1 | 2 | 1 |
| $r$ | 1 | 1 | 1 | 2 | 1 | 1 | 2 |
| $s$ | 1 | 2 | 1 | 1 | 3 | 2 | 2 |
| $a$ | 116 | 172 | 212 | 232 | 236 | 292 | 308 |

A leapfrog fullerene when $r \equiv s \equiv 0 \pmod 3$

$T$ | 12 | $\mathcal{B}_6$



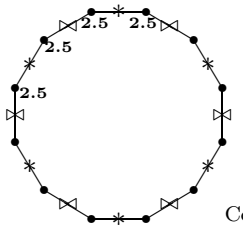

Coordinate codes:
● $(p, p+r)$
◇ $(p+s, p+r)$

Atoms:
$a = 4(15p^2 + 5r^2 + s^2 + 15pr + 9ps + 4rs)$

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| $p$ | 1 | 1 | 1 | 1 | 1 | 1 | 2 |
| $r$ | 1 | 1 | 2 | 1 | 2 | 1 | 1 |
| $s$ | 1 | 2 | 1 | 3 | 2 | 4 | 1 |
| $a$ | 196 | 260 | 332 | 332 | 412 | 412 | 472 |

A leapfrog fullerene when $r \equiv s \equiv 0 \pmod 3$

## $D_{6h}$ 24 $\mathcal{D}_1$



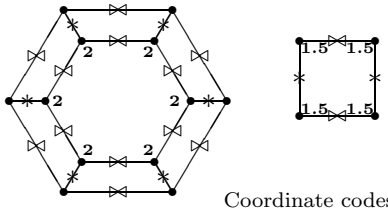

Coordinate codes:
$\bowtie$ $(p,p)$
$*$ $(r)$

Atoms:
$a = 36p^2 + 48pr + 12r^2$

| $p$ | 1 | 1 | 2 | 1 | 2 | 1 | 3 |
|---|---|---|---|---|---|---|---|
| $r$ | 1 | 2 | 1 | 3 | 2 | 4 | 1 |
| $a$ | 96 | 180 | 252 | 288 | 384 | 420 | 480 |

A leapfrog fullerene when $r \equiv 0 \pmod 3$

## $D_{6h}$ 24 Nanotube parameter: $r$ $\mathcal{E}_1$



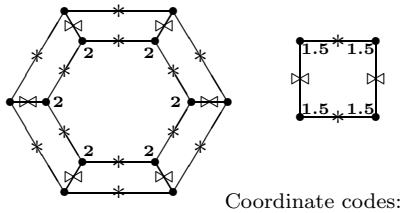

Coordinate codes:
$\bowtie$ $(p,p)$
$*$ $(r)$

Atoms:
$a = 36p^2 + 24pr$

| $p$ | 1 | 1 | 1 | 1 | 1 | 1 | 2 |
|---|---|---|---|---|---|---|---|
| $r$ | 1 | 2 | 3 | 4 | 5 | 6 | 1 |
| $a$ | 60 | 84 | 108 | 132 | 156 | 180 | 192 |

A leapfrog fullerene when $r \equiv 0 \pmod 3$

## $D_{6h}$ 24 Nanotube parameter: $p$ $\mathcal{E}_2$



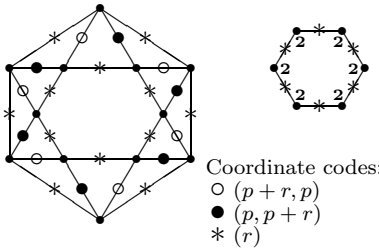

Coordinate codes:
$\bowtie$ $(p,p)$
$*$ $(r)$

Atoms:
$a = 24pr + 12r^2$

| $p$ | 1 | 2 | 3 | 1 | 4 | 5 | 2 |
|---|---|---|---|---|---|---|---|
| $r$ | 1 | 1 | 1 | 2 | 1 | 1 | 2 |
| $a$ | 36 | 60 | 84 | 96 | 108 | 132 | 144 |

A leapfrog fullerene when $r \equiv 0 \pmod 3$

## $D_{6d}$ 24 Nanotube parameter: $p$ $\mathcal{F}_4$



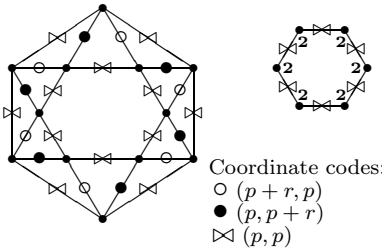

Coordinate codes:
$\circ$ $(p+r,p)$
$\bullet$ $(p,p+r)$
$*$ $(r)$

Atoms:
$a = 24pr + 24r^2$

| $p$ | 1 | 2 | 3 | 4 | 1 | 5 | 6 |
|---|---|---|---|---|---|---|---|
| $r$ | 1 | 1 | 1 | 1 | 2 | 1 | 1 |
| $a$ | 48 | 72 | 96 | 120 | 144 | 144 | 168 |

A leapfrog fullerene when $r \equiv 0 \pmod 3$

## $D_{6d}$ 24 Nanotube parameter: $r$ $\mathcal{F}_5$



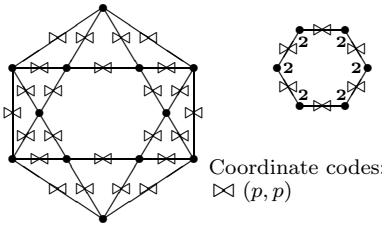

Coordinate codes:
$\circ$ $(p+r,p)$
$\bullet$ $(p,p+r)$
$\bowtie$ $(p,p)$

Atoms:
$a = 72p^2 + 24pr$

| $p$ | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
|---|---|---|---|---|---|---|---|
| $r$ | 1 | 2 | 3 | 4 | 5 | 6 | 7 |
| $a$ | 96 | 120 | 144 | 168 | 192 | 216 | 240 |

A leapfrog fullerene when $r \equiv 0 \pmod 3$

## $D_{6d}$ 24 $\mathcal{F}_2$



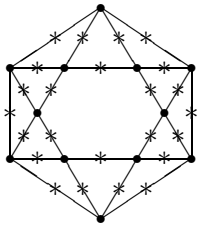

Coordinate codes:
$\bowtie$ $(p,p)$

Atoms:
$a = 72p^2$

| $p$ | 1 | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|---|
| $a$ | 72 | 288 | 648 | 1152 | 1800 | 2592 |

All are leapfrog fullerenes

## $D_{6d}$ 24 $\mathcal{F}_1$




Coordinate codes:
$*$ $(r)$

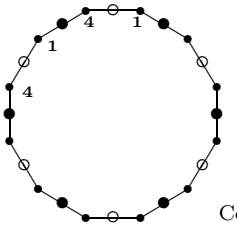
Atoms:

$a = 24r^2$

| $r$ | 1 | 2 | 3 | 4 | 5 | 6 | 7 |
|---|---|---|---|---|---|---|---|
| $a$ | 24 | 96 | 216 | 384 | 600 | 864 | 1176 |

A leapfrog fullerene when $r \equiv 0 \pmod 3$

## $D_{6d}$ 24 $\mathcal{D}_2$




Coordinate codes:
$\circ$ $(p+r,p)$
$\bullet$ $(p,p+r)$

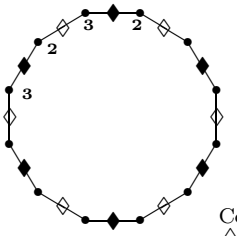
Atoms:

$a = 24(3p^2 + 5pr + 2r^2)$

| $p$ | 1 | 1 | 2 | 1 | 2 | 3 | 1 |
|---|---|---|---|---|---|---|---|
| $r$ | 1 | 2 | 1 | 3 | 2 | 1 | 4 |
| $a$ | 240 | 504 | 576 | 864 | 960 | 1056 | 1320 |

A leapfrog fullerene when $r \equiv 0 \pmod 3$

## $D_{6d}$ 24 $\mathcal{D}_3$




Coordinate codes:
$\Diamond$ $(p,q)$
$\blacklozenge$ $(q,p)$

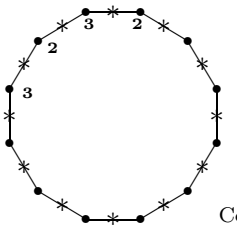
Atoms:

$a = 24(p^2 + 3pq + 2q^2)$

| $p$ | 1 | 2 | 1 | 3 | 2 | 1 | 4 |
|---|---|---|---|---|---|---|---|
| $q$ | 1 | 1 | 2 | 1 | 2 | 3 | 1 |
| $a$ | 144 | 288 | 360 | 480 | 576 | 672 | 720 |

A leapfrog fullerene when $p \equiv q \pmod 3$

## $D_{6d}$ 24 $\mathcal{D}_4$




Coordinate codes:
$*$ $(r)$

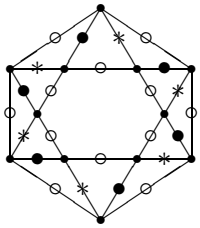
Atoms:

$a = 48r^2$

| $r$ | 1 | 2 | 3 | 4 | 5 | 6 | 7 |
|---|---|---|---|---|---|---|---|
| $a$ | 48 | 192 | 432 | 768 | 1200 | 1728 | 2352 |

A leapfrog fullerene when $r \equiv 0 \pmod 3$

## $D_6$ 12 $\mathcal{F}_6$




Coordinate codes:
$\circ$ $(p+r,p)$
$\bullet$ $(p,p+r)$
$*$ $(r)$

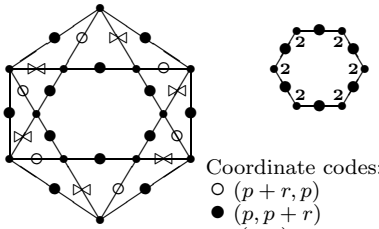
Atoms:

$a = 36p^2 + 60pr + 24r^2$

| $p$ | 1 | 1 | 2 | 1 | 2 | 3 | 1 |
|---|---|---|---|---|---|---|---|
| $r$ | 1 | 2 | 1 | 3 | 2 | 1 | 4 |
| $a$ | 120 | 252 | 288 | 432 | 480 | 528 | 660 |

A leapfrog fullerene when $r \equiv 0 \pmod 3$

## $D_6$ 12 $\mathcal{F}_7$




Coordinate codes:
$\circ$ $(p+r,p)$
$\bullet$ $(p,p+r)$
$\bowtie$ $(p,p)$

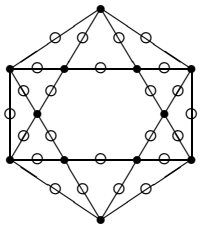
Atoms:

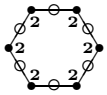
$a = 72p^2 + 60pr + 12r^2$

| $p$ | 1 | 1 | 1 | 2 | 1 | 2 | 1 |
|---|---|---|---|---|---|---|---|
| $r$ | 1 | 2 | 3 | 1 | 4 | 2 | 5 |
| $a$ | 144 | 240 | 360 | 420 | 504 | 576 | 672 |

A leapfrog fullerene when $r \equiv 0 \pmod 3$

## $D_6$ | 12 | | $\mathcal{F}_3$




Coordinate codes:
○ $(p+r, p)$

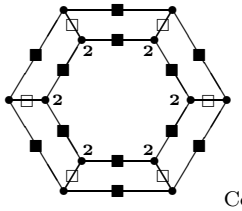
Atoms:

$a = 72p^2 + 72pr + 24r^2$

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| $p$ | 1 | 1 | 2 | 1 | 2 | 1 | 3 |
| $r$ | 1 | 2 | 1 | 3 | 2 | 4 | 1 |
| $a$ | 168 | 312 | 456 | 504 | 672 | 744 | 888 |

A leapfrog fullerene when $r \equiv 0 \pmod 3$

## $D_6$ | 12 | Nanotube parameter: $r$ | $\mathcal{E}_3$





Coordinate codes:
□ $(p+r, q)$
■ $(p, q+s)$

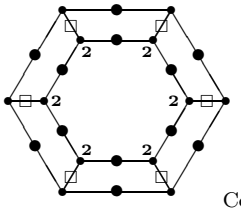
Atoms:

$a = 12(2p^2 + 2q^2 + s^2 + 2pq + 2ps + 3qs + r(p+q+s))$

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| $p$ | 1 | 1 | 1 | 1 | 1 | 2 | 1 |
| $q$ | 1 | 1 | 1 | 1 | 1 | 1 | 2 |
| $r$ | 1 | 2 | 3 | 1 | 4 | 1 | 1 |
| $s$ | 1 | 1 | 1 | 2 | 1 | 1 | 1 |
| $a$ | 180 | 216 | 252 | 288 | 288 | 312 | 324 |

Leapfrog fullerene when $p - q \equiv -r \equiv s \pmod 3$

## $D_6$ | 12 | Nanotube parameter: $s$ | $\mathcal{E}_4$





Coordinate codes:
● $(p, p+r)$
□ $(p+s, p+r)$

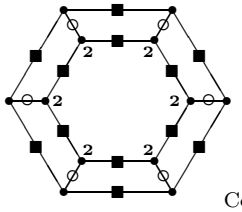
Atoms:

$a = 12(6p^2 + 6pr + 2r^2 + s(2p+r))$

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| $p$ | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| $r$ | 1 | 1 | 1 | 1 | 1 | 2 | 1 |
| $s$ | 1 | 2 | 3 | 4 | 5 | 1 | 6 |
| $a$ | 204 | 240 | 276 | 312 | 348 | 360 | 384 |

A leapfrog fullerene when $r \equiv s \equiv 0 \pmod 3$

## $D_6$ | 12 | | $\mathcal{E}_5$





Coordinate codes:
○ $(p+r, p)$
■ $(p+r, p+s)$

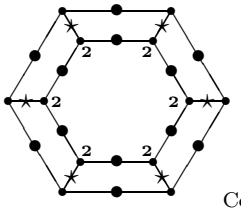
Atoms:

$a = 12(6p^2 + 2r^2 + s^2 + 6pr + 5ps + 2rs)$

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| $p$ | 1 | 1 | 1 | 1 | 1 | 2 | 1 |
| $r$ | 1 | 1 | 2 | 1 | 2 | 1 | 3 |
| $s$ | 1 | 2 | 1 | 3 | 2 | 1 | 1 |
| $a$ | 264 | 384 | 432 | 528 | 576 | 612 | 648 |

A leapfrog fullerene when $r \equiv s \equiv 0 \pmod 3$

## $D_6$ | 12 | Nanotube parameter: $s$ | $\mathcal{E}_6$





Coordinate codes:
● $(p, p+r)$
★ $(r+s)$

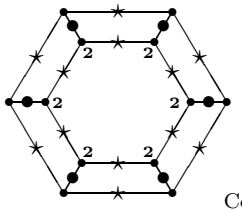
Atoms:

$a = 12(3p^2 + 2r^2 + 5pr + s(2p+r))$

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| $p$ | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| $r$ | 1 | 1 | 1 | 1 | 1 | 2 | 1 |
| $s$ | 1 | 2 | 3 | 4 | 5 | 1 | 6 |
| $a$ | 156 | 192 | 228 | 264 | 300 | 300 | 336 |

A leapfrog fullerene when $r \equiv 0 \pmod 3$

## $D_6$ | 12 | Nanotube parameter: $p$ | $\mathcal{E}_7$





Coordinate codes:
● $(p, p+r)$
★ $(r+s)$

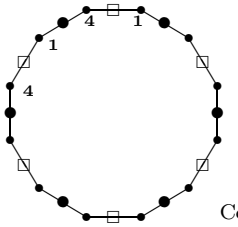
Atoms:

$a = 12(2r^2 + s^2 + 3rs + 2p(r+s))$

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| $p$ | 1 | 2 | 1 | 3 | 1 | 4 | 2 |
| $r$ | 1 | 1 | 1 | 1 | 2 | 1 | 1 |
| $s$ | 1 | 1 | 2 | 1 | 1 | 1 | 2 |
| $a$ | 120 | 168 | 216 | 216 | 252 | 264 | 288 |

A leapfrog fullerene when $r \equiv 0 \pmod 3$

## $D_6$ | 12 | $\mathcal{D}_5$




Coordinate codes:
□ $(p+r, p+q)$
● $(p, p+q+s)$

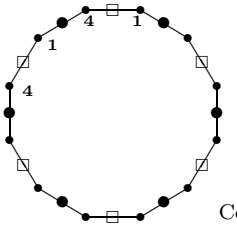
Atoms:

$a = 12(6p^2 + 2q^2 + r^2 + s^2 + 6pq + 5pr + 5ps + 3qr + 2qs + 2rs)$

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| $p$ | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| $q$ | 1 | 1 | 1 | 2 | 1 | 1 | 1 |
| $r$ | 1 | 1 | 2 | 1 | 1 | 2 | 3 |
| $s$ | 1 | 2 | 1 | 1 | 3 | 2 | 1 |
| $a$ | 396 | 540 | 552 | 600 | 708 | 720 | 732 |

A leapfrog fullerene when $q \equiv r \equiv -s \pmod 3$

## $D_6$ | 12 | $\mathcal{D}_6$




Coordinate codes:
□ $(p+r, p)$
● $(p, p+r+s)$

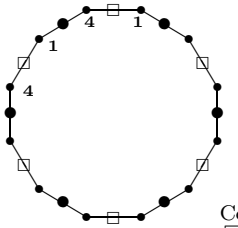
Atoms:

$a = 12(6p^2 + 4r^2 + s^2 + 10pr + 5ps + 4rs)$

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| $p$ | 1 | 1 | 1 | 1 | 2 | 1 | 1 |
| $r$ | 1 | 1 | 1 | 2 | 1 | 2 | 1 |
| $s$ | 1 | 2 | 3 | 1 | 1 | 2 | 4 |
| $a$ | 360 | 504 | 672 | 672 | 756 | 864 | 864 |

A leapfrog fullerene when $r \equiv s \equiv 0 \pmod 3$

## $D_6$ | 12 | $\mathcal{D}_7$




Coordinate codes:
□ $(p+s, p+r)$
● $(p, p+r)$

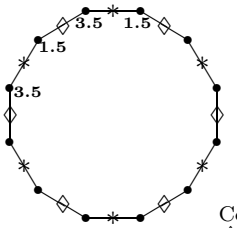
Atoms:

$a = 12(6p^2 + 2r^2 + s^2 + 6pr + 5ps + 3rs)$

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| $p$ | 1 | 1 | 1 | 1 | 1 | 2 | 1 |
| $r$ | 1 | 1 | 2 | 1 | 2 | 1 | 3 |
| $s$ | 1 | 2 | 1 | 3 | 2 | 1 | 1 |
| $a$ | 276 | 408 | 456 | 564 | 624 | 624 | 684 |

A leapfrog fullerene when $r \equiv s \equiv 0 \pmod 3$

## $D_6$ | 12 | $\mathcal{D}_8$




Coordinate codes:
◊ $(p, q)$
∗ $(p+r)$

$r \geq 0$

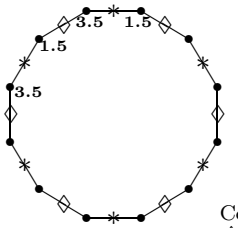
Atoms:

$a = 24p^2 + 12q^2 + 12r^2 + 36pq + 24pr + 24qr$

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| $p$ | 1 | 1 | 1 | 2 | 1 | 1 | 1 |
| $q$ | 1 | 1 | 2 | 1 | 1 | 2 | 3 |
| $r$ | 0 | 1 | 0 | 0 | 2 | 1 | 0 |
| $a$ | 72 | 132 | 144 | 180 | 216 | 228 | 240 |

A leapfrog fullerene when $p \equiv q \equiv -r \pmod 3$

## $D_6$ | 12 | $\mathcal{D}_9$




Coordinate codes:
◊ $(p+r, p+s)$
∗ $(r)$

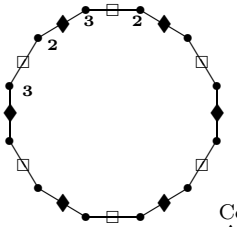
Atoms:

$a = 12(3p^2 + 2r^2 + s^2 + 5pr + 3ps + 3rs)$

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| $p$ | 1 | 1 | 1 | 2 | 1 | 1 | 2 |
| $r$ | 1 | 1 | 2 | 1 | 1 | 2 | 1 |
| $s$ | 1 | 2 | 1 | 1 | 3 | 2 | 2 |
| $a$ | 204 | 312 | 372 | 408 | 444 | 516 | 552 |

A leapfrog fullerene when $r \equiv s \equiv 0 \pmod 3$

## $D_6$ | 12 | $\mathcal{D}_{10}$




Coordinate codes:
♦ $(p, q+r)$
□ $(q, p)$

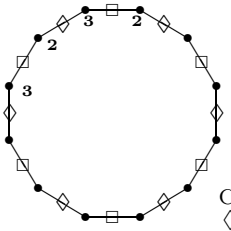
Atoms:

$a = 12(2p^2 + 4q^2 + r^2 + 6pq + 3pr + 4qr)$

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| $p$ | 1 | 1 | 2 | 1 | 1 | 2 | 3 |
| $q$ | 1 | 1 | 1 | 2 | 1 | 1 | 1 |
| $r$ | 1 | 2 | 1 | 1 | 3 | 2 | 1 |
| $a$ | 240 | 360 | 420 | 504 | 504 | 576 | 648 |

A leapfrog fullerene when $p \equiv q$ & $r \equiv 0 \pmod 3$

## $D_6$ | 12 | $\mathcal{D}_{11}$



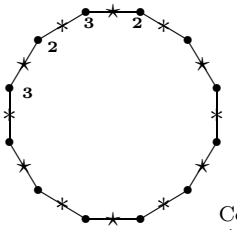

Coordinate codes:
◊ $(p+s,q)$
□ $(r,p)$

Atoms:
$a = 12(2p^2+q^2+r^2+s^2+3pq+3pr+2ps+2qr+qs+2rs)$

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| $p$ | 1 | 1 | 1 | 1 | 2 | 1 | 1 |
| $q$ | 1 | 1 | 2 | 1 | 1 | 1 | 2 |
| $r$ | 1 | 1 | 1 | 2 | 1 | 1 | 1 |
| $s$ | 1 | 2 | 1 | 1 | 1 | 3 | 2 |
| $a$ | 216 | 312 | 324 | 336 | 384 | 432 | 432 |

A leapfrog fullerene when $p \equiv r \equiv q - s \pmod 3$

## $D_6$ | 12 | $\mathcal{D}_{12}$



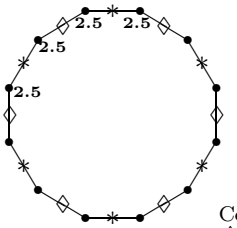

Coordinate codes:
∗ $(r+s)$
★ $(s)$

Atoms:
$a = 12(r^2+4s^2+4rs)$

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| $r$ | 1 | 2 | 1 | 3 | 2 | 4 | 1 |
| $s$ | 1 | 1 | 2 | 1 | 2 | 1 | 3 |
| $a$ | 108 | 192 | 300 | 300 | 432 | 432 | 588 |

A leapfrog fullerene when $r \equiv s \equiv 0 \pmod 3$

## $D_6$ | 12 | $\mathcal{D}_{13}$



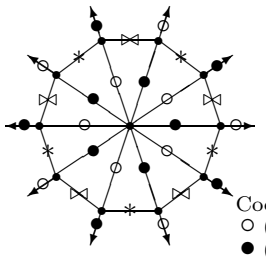

Coordinate codes:
◊ $(p+s,p)$
∗ $(r)$

Atoms:
$a = 12(3p^2+r^2+s^2+4pr+3ps+2rs)$

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| $p$ | 1 | 1 | 1 | 2 | 1 | 1 | 1 |
| $r$ | 1 | 1 | 2 | 1 | 1 | 2 | 3 |
| $s$ | 1 | 2 | 1 | 1 | 3 | 2 | 1 |
| $a$ | 168 | 264 | 276 | 360 | 384 | 396 | 408 |

A leapfrog fullerene when $r \equiv s \equiv 0 \pmod 3$

## $D_{5h}$ | 20 | $\mathcal{G}_1$



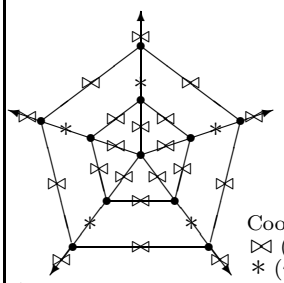
Coordinate codes:
○ $(p+r,p)$
● $(p,p+r)$
⋈ $(p,p)$
∗ $(r)$

Atoms:
$a = 30p^2+40pr+10r^2$

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| $p$ | 1 | 1 | 2 | 1 | 2 | 1 | 3 |
| $r$ | 1 | 2 | 1 | 3 | 2 | 4 | 1 |
| $a$ | 80 | 150 | 210 | 240 | 320 | 350 | 400 |

A leapfrog fullerene when $r \equiv 0 \pmod 3$

## $D_{5h}$ | 20 | Nanotube parameter: $r$ | $\mathcal{H}_1$



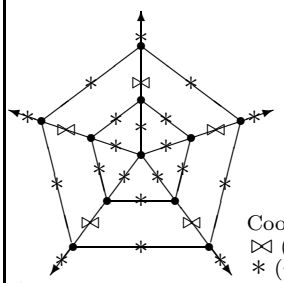

Coordinate codes:
⋈ $(p,p)$
∗ $(r)$

Atoms:
$a = 30p^2+20pr$

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| $p$ | 1 | 1 | 1 | 1 | 1 | 1 | 2 |
| $r$ | 1 | 2 | 3 | 4 | 5 | 6 | 1 |
| $a$ | 50 | 70 | 90 | 110 | 130 | 150 | 160 |

A leapfrog fullerene when $r \equiv 0 \pmod 3$

## $D_{5h}$ | 20 | Nanotube parameter: $p$ | $\mathcal{H}_2$



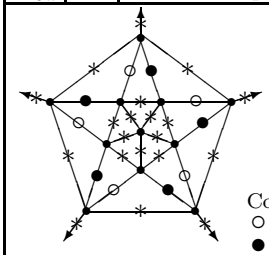

Coordinate codes:
⋈ $(p,p)$
∗ $(r)$

Atoms:
$a = 20pr+10r^2$

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| $p$ | 1 | 2 | 3 | 1 | 4 | 5 | 2 | 6 |
| $r$ | 1 | 1 | 1 | 2 | 1 | 1 | 2 | 1 |
| $a$ | 30 | 50 | 70 | 80 | 90 | 110 | 120 | 130 |

A leapfrog fullerene when $r \equiv 0 \pmod 3$

## $D_{5d}$ | 20 | Nanotube parameter: $p$ | $\mathcal{A}_4$



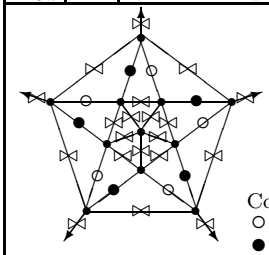
Coordinate codes:
- ○ $(p+r,p)$
- ● $(p,p+r)$
- ∗ $(r)$

Atoms:
$a = 20pr + 20r^2$

| $p$ | 1 | 2 | 3 | 4 | 1 | 5 | 6 |
|---|---|---|---|---|---|---|---|
| $r$ | 1 | 1 | 1 | 1 | 2 | 1 | 1 |
| $a$ | 40 | 60 | 80 | 100 | 120 | 120 | 140 |

A leapfrog fullerene when $r \equiv 0 \pmod 3$

## $D_{5d}$ | 20 | Nanotube parameter: $r$ | $\mathcal{A}_5$



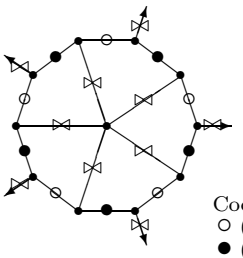
Coordinate codes:
- ○ $(p+r,p)$
- ● $(p,p+r)$
- ⋈ $(p,p)$

Atoms:
$a = 60p^2 + 20pr$

| $p$ | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
|---|---|---|---|---|---|---|---|
| $r$ | 1 | 2 | 3 | 4 | 5 | 6 | 7 |
| $a$ | 80 | 100 | 120 | 140 | 160 | 180 | 200 |

A leapfrog fullerene when $r \equiv 0 \pmod 3$

## $D_{5d}$ | 20 | | $\mathcal{I}_1$



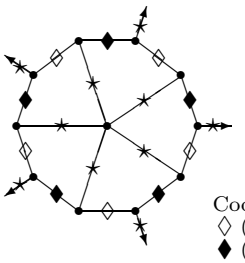

Coordinate codes:
- ○ $(p+r,p)$
- ● $(p,p+r)$
- ⋈ $(p+r,p+r)$

Atoms:
$a = 20(3p^2 + 2r^2 + 5pr)$

| $p$ | 1 | 1 | 2 | 1 | 2 | 3 | 1 |
|---|---|---|---|---|---|---|---|
| $r$ | 1 | 2 | 1 | 3 | 2 | 1 | 4 |
| $a$ | 200 | 420 | 480 | 720 | 800 | 880 | 1100 |

A leapfrog fullerene when $r \equiv 0 \pmod 3$

## $D_{5d}$ | 20 | | $\mathcal{I}_2$



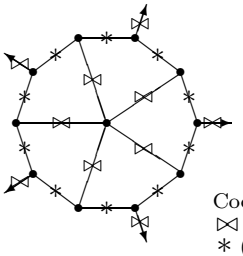

Coordinate codes:
- ◇ $(p,q)$
- ◆ $(q,p)$
- ★ $(p+2q)$

Atoms:
$a = 20(p^2 + 3pq + 2q^2)$

| $p$ | 1 | 2 | 1 | 3 | 2 | 1 | 4 |
|---|---|---|---|---|---|---|---|
| $q$ | 1 | 1 | 2 | 1 | 2 | 3 | 1 |
| $a$ | 120 | 240 | 300 | 400 | 480 | 560 | 600 |

A leapfrog fullerene when $p \equiv q \pmod 3$

## $D_{5d}$ | 20 | | $\mathcal{I}_3$



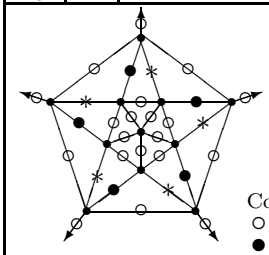

Coordinate codes:
- ⋈ $(r,r)$
- ∗ $(r)$

Atoms:
$a = 40r^2$

| $r$ | 1 | 2 | 3 | 4 | 5 | 6 | 7 |
|---|---|---|---|---|---|---|---|
| $a$ | 40 | 160 | 360 | 640 | 1000 | 1440 | 1960 |

A leapfrog fullerene when $r \equiv 0 \pmod 3$

## $D_5$ | 10 | | $\mathcal{A}_6$



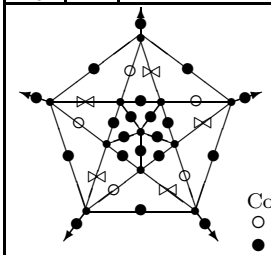
Coordinate codes:
- ○ $(p+r,p)$
- ● $(p,p+r)$
- ∗ $(r)$

Atoms:
$a = 30p^2 + 50pr + 20r^2$

| $p$ | 1 | 1 | 2 | 1 | 2 | 3 | 1 |
|---|---|---|---|---|---|---|---|
| $r$ | 1 | 2 | 1 | 3 | 2 | 1 | 4 |
| $a$ | 100 | 210 | 240 | 360 | 400 | 440 | 550 |

A leapfrog fullerene when $r \equiv 0 \pmod 3$

## $D_5$ | 10 | | $\mathcal{A}_7$



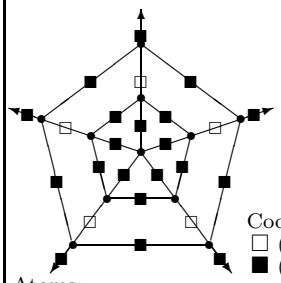
Coordinate codes:
○ $(p+r,p)$
● $(p,p+r)$
⋈ $(p,p)$

Atoms:
$a = 60p^2 + 50pr + 10r^2$

| $p$ | 1 | 1 | 1 | 2 | 1 | 2 | 1 |
|---|---|---|---|---|---|---|---|
| $r$ | 1 | 2 | 3 | 1 | 4 | 2 | 5 |
| $a$ | 120 | 200 | 300 | 350 | 420 | 480 | 560 |

A leapfrog fullerene when $r \equiv 0 \pmod 3$

## $D_5$ | 10 | Nanotube parameter: $r$ | $\mathcal{H}_3$



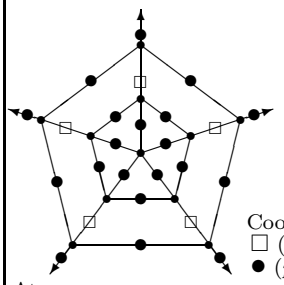

Coordinate codes:
□ $(p+r,q)$
■ $(p,q+s)$

Atoms:
$a = 10(2p^2 + 2q^2 + s^2 + 2pq + 2ps + 3qs + r(p+q+s))$

| $p$ | 1 | 1 | 1 | 1 | 1 | 1 | 2 | 1 |
|---|---|---|---|---|---|---|---|---|
| $q$ | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 2 |
| $r$ | 1 | 2 | 3 | 1 | 4 | 1 | 1 | 1 |
| $s$ | 1 | 1 | 1 | 2 | 1 | 1 | 1 | 1 |
| $a$ | 150 | 180 | 210 | 240 | 240 | 260 | 270 | |

Leapfrog fullerene when $p - q \equiv -r \equiv s \pmod 3$

## $D_5$ | 10 | Nanotube parameter: $s$ | $\mathcal{H}_4$



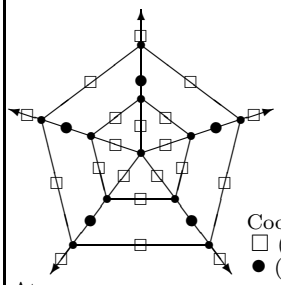

Coordinate codes:
□ $(p+s,p+r)$
● $(p,p+r)$

Atoms:
$a = 10(6p^2 + 6pr + 2r^2 + s(2p+r))$

| $p$ | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
|---|---|---|---|---|---|---|---|
| $r$ | 1 | 1 | 1 | 1 | 1 | 2 | 1 |
| $s$ | 1 | 2 | 3 | 4 | 5 | 1 | 6 |
| $a$ | 170 | 200 | 230 | 260 | 290 | 300 | 320 |

A leapfrog fullerene when $r \equiv s \equiv 0 \pmod 3$

## $D_5$ | 10 | | $\mathcal{H}_5$



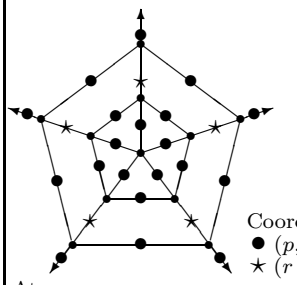

Coordinate codes:
□ $(p+s,p+r)$
● $(p,p+r)$

Atoms:
$a = 10(6p^2 + 2r^2 + s^2 + 6pr + 5ps + 2rs)$

| $p$ | 1 | 1 | 1 | 1 | 1 | 2 | 1 |
|---|---|---|---|---|---|---|---|
| $r$ | 1 | 1 | 2 | 1 | 2 | 1 | 3 |
| $s$ | 1 | 2 | 1 | 3 | 2 | 1 | 1 |
| $a$ | 220 | 320 | 360 | 440 | 480 | 510 | 540 |

A leapfrog fullerene when $r \equiv s \equiv 0 \pmod 3$

## $D_5$ | 10 | Nanotube parameter: $s$ | $\mathcal{H}_6$



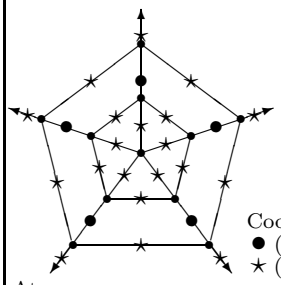

Coordinate codes:
● $(p,p+r)$
★ $(r+s)$

Atoms:
$a = 10(3p^2 + 2r^2 + 5pr + s(2p+r))$

| $p$ | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
|---|---|---|---|---|---|---|---|
| $r$ | 1 | 1 | 1 | 1 | 1 | 2 | 1 |
| $s$ | 1 | 2 | 3 | 4 | 5 | 1 | 6 |
| $a$ | 130 | 160 | 190 | 220 | 250 | 250 | 280 |

A leapfrog fullerene when $r \equiv s \equiv 0 \pmod 3$

## $D_5$ | 10 | Nanotube parameter: $p$ | $\mathcal{H}_7$



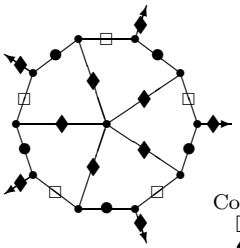

Coordinate codes:
● $(p,p+r)$
★ $(r+s)$

Atoms:
$a = 10(2r^2 + s^2 + 3rs + s + 2p(r+s))$

| $p$ | 1 | 2 | 1 | 3 | 1 | 4 | 2 |
|---|---|---|---|---|---|---|---|
| $r$ | 1 | 1 | 1 | 1 | 2 | 1 | 1 |
| $s$ | 1 | 1 | 2 | 1 | 1 | 1 | 2 |
| $a$ | 100 | 140 | 180 | 180 | 210 | 220 | 240 |

A leapfrog fullerene when $r \equiv s \equiv 0 \pmod 3$

## $D_5$ 10 $\mathcal{I}_4$



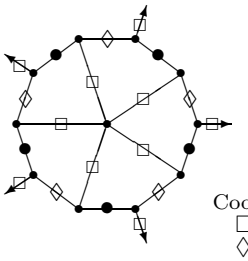

Coordinate codes:
□ $(p+r, p+q)$
♦ $(p+s, p+q+r)$
● $(p, p+q+s)$

Atoms:
$a = 10(6p^2 + 2q^2 + r^2 + s^2 + 6pq + 5pr + 5ps + 3qr + 2qs + 2rs)$

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| $p$ | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| $q$ | 1 | 1 | 1 | 2 | 1 | 1 | 1 |
| $r$ | 1 | 1 | 2 | 1 | 1 | 2 | 3 |
| $s$ | 1 | 2 | 1 | 1 | 3 | 2 | 1 |
| $a$ | 330 | 450 | 460 | 500 | 590 | 600 | 610 |

A leapfrog fullerene when $q \equiv r \equiv -s \pmod 3$

## $D_5$ 10 $\mathcal{I}_5$



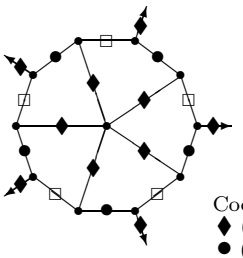

Coordinate codes:
□ $(p+s, p+r)$
◊ $(p+r, p)$
● $(p, p+r+s)$

Atoms:
$a = 10(6p^2 + 4r^2 + s^2 + 10pr + 5ps + 4rs)$

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| $p$ | 1 | 1 | 1 | 1 | 2 | 1 | 1 |
| $r$ | 1 | 1 | 1 | 2 | 1 | 2 | 1 |
| $s$ | 1 | 2 | 3 | 1 | 1 | 2 | 4 |
| $a$ | 300 | 420 | 560 | 560 | 630 | 720 | 720 |

A leapfrog fullerene when $r \equiv s \equiv 0 \pmod 3$

## $D_5$ 10 $\mathcal{I}_6$



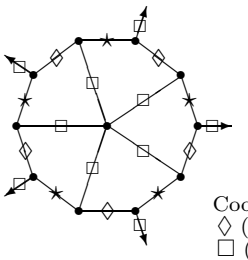

Coordinate codes:
♦ $(p, p+r+s)$
● $(p, p+r)$
□ $(p+s, p+r)$

Atoms:
$a = 10(6p^2 + 2r^2 + s^2 + 6pr + 5ps + 3rs)$

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| $p$ | 1 | 1 | 1 | 1 | 1 | 2 | 1 |
| $r$ | 1 | 1 | 2 | 1 | 2 | 1 | 3 |
| $s$ | 1 | 2 | 1 | 3 | 2 | 1 | 1 |
| $a$ | 230 | 340 | 380 | 470 | 520 | 520 | 570 |

A leapfrog fullerene when $r \equiv s \equiv 0 \pmod 3$

## $D_5$ 10 $\mathcal{I}_7$



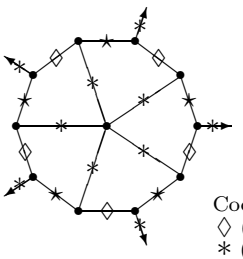

Coordinate codes:
◊ $(p, q)$
□ $(p+q, r)$
★ $(p+r)$

Atoms:
$a = 10(2p^2 + q^2 + r^2 + 3pq + 2pr + 2qr)$

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| $p$ | 1 | 1 | 1 | 2 | 1 | 1 | 1 |
| $q$ | 1 | 1 | 2 | 1 | 1 | 2 | 3 |
| $r$ | 1 | 2 | 1 | 1 | 3 | 2 | 1 |
| $a$ | 110 | 180 | 190 | 220 | 270 | 280 | 290 |

A leapfrog fullerene when $p \equiv q \equiv -r \pmod 3$

## $D_5$ 10 $\mathcal{I}_8$



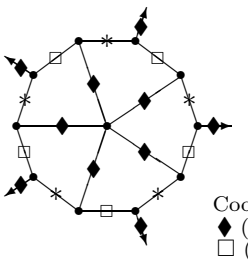

Coordinate codes:
◊ $(p, q)$
∗ $(p+q)$
★ $(p)$

Atoms:
$a = 10(2p^2 + q^2 + 3pq)$

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| $p$ | 1 | 1 | 2 | 1 | 2 | 3 | 1 |
| $q$ | 1 | 2 | 1 | 3 | 2 | 1 | 4 |
| $a$ | 60 | 120 | 150 | 200 | 240 | 280 | 300 |

A leapfrog fullerene when $p \equiv q \equiv 0 \pmod 3$

## $D_5$ 10 $\mathcal{I}_9$



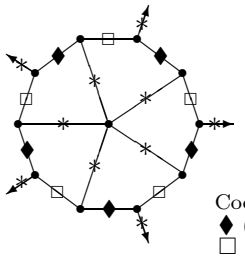

Coordinate codes:
♦ $(p, p+r+s)$
□ $(p+r, p+s)$
∗ $(r)$

Atoms:
$a = 10(3p^2 + 2r^2 + s^2 + 5pr + 3ps + 3rs)$

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| $p$ | 1 | 1 | 1 | 2 | 1 | 1 | 2 |
| $r$ | 1 | 1 | 2 | 1 | 1 | 2 | 1 |
| $s$ | 1 | 2 | 1 | 1 | 3 | 2 | 2 |
| $a$ | 170 | 260 | 310 | 340 | 370 | 430 | 460 |

A leapfrog fullerene when $r \equiv s \equiv 0 \pmod 3$

## $D_5$ | 10 | $\mathcal{I}_{10}$



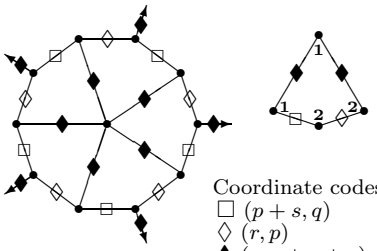

Coordinate codes:
♦ $(p, q+r)$
□ $(q, p)$
∗ $(p+2q+r)$

Atoms:
$a = 10(2p^2 + 4q^2 + r^2 + 6pq + 3pr + 4qr)$

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| $p$ | 1 | 1 | 2 | 1 | 1 | 2 | 3 |
| $q$ | 1 | 1 | 1 | 2 | 1 | 1 | 1 |
| $r$ | 1 | 2 | 1 | 1 | 3 | 2 | 1 |
| $a$ | 200 | 300 | 350 | 420 | 420 | 480 | 540 |

A leapfrog fullerene when $p \equiv q \equiv r \pmod 3$

## $D_5$ | 10 | $\mathcal{I}_{11}$



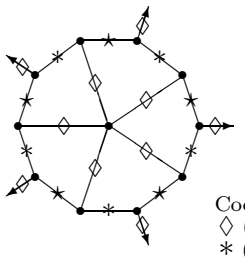

Coordinate codes:
□ $(p+s, q)$
♢ $(r, p)$
♦ $(s, p+q+r)$

Atoms:
$a = 10(2p^2 + q^2 + r^2 + s^2 + 3pq + 3pr + 2ps + 2qr + qs + 2rs)$

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| $p$ | 1 | 1 | 1 | 1 | 2 | 1 | 1 |
| $q$ | 1 | 1 | 2 | 1 | 1 | 1 | 2 |
| $r$ | 1 | 1 | 1 | 2 | 1 | 1 | 1 |
| $s$ | 1 | 2 | 1 | 1 | 1 | 3 | 2 |
| $a$ | 180 | 260 | 270 | 280 | 320 | 360 | 360 |

A leapfrog fullerene when $p \equiv r \equiv q - s \pmod 3$

## $D_5$ | 10 | $\mathcal{I}_{12}$



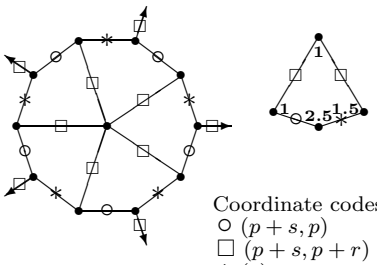

Coordinate codes:
♢ $(r+s, s)$
∗ $(r+s)$
★ $(s)$

Atoms:
$a = 10(r^2 + 4rs + 4s^2)$

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| $r$ | 1 | 2 | 1 | 3 | 2 | 4 | 1 |
| $s$ | 1 | 1 | 2 | 1 | 2 | 1 | 3 |
| $a$ | 90 | 160 | 250 | 250 | 360 | 360 | 490 |

A leapfrog fullerene when $r \equiv s \equiv 0 \pmod 3$

## $D_5$ | 10 | $\mathcal{I}_{13}$



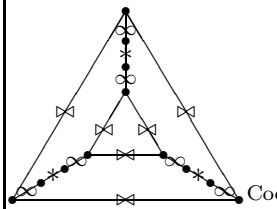

Coordinate codes:
○ $(p+s, p)$
□ $(p+s, p+r)$
∗ $(r)$

Atoms:
$a = 10(3p^2 + r^2 + s^2 + 4pr + 3ps + 2rs)$

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| $p$ | 1 | 1 | 1 | 2 | 1 | 1 | 1 |
| $r$ | 1 | 1 | 2 | 1 | 1 | 2 | 3 |
| $s$ | 1 | 2 | 1 | 1 | 3 | 2 | 1 |
| $a$ | 140 | 220 | 230 | 300 | 320 | 330 | 340 |

A leapfrog fullerene when $r \equiv s \equiv 0 \pmod 3$

## $D_{3h}$ | 12 | Nanotube parameter: $r$ | $\mathcal{J}_1$



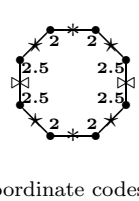

Coordinate codes:
⋈ $(p, p)$
∞ $(q, q)$
∗ $(r)$

Atoms:
$a = 6p^2 + 18q^2 + 36pq + 12r(p+q)$

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| $p$ | 1 | 1 | 1 | 2 | 1 | 1 | 2 |
| $q$ | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| $r$ | 1 | 2 | 3 | 1 | 4 | 5 | 2 |
| $a$ | 84 | 108 | 132 | 150 | 156 | 180 | 186 |

A leapfrog fullerene when $r \equiv 0 \pmod 3$

## $D_{3h}$ | 12 | Nanotube parameter: $p$ | $\mathcal{J}_2$



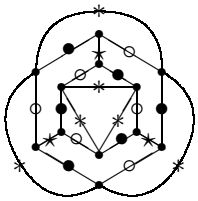

Coordinate codes:
⋈ $(p, p)$
∗ $(r)$
★ $(s)$

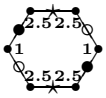
Atoms:
$a = 2r^2 + 6s^2 + 12rs + 12p(r+s)$

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| $p$ | 1 | 2 | 1 | 1 | 3 | 1 | 2 |
| $r$ | 1 | 1 | 2 | 1 | 1 | 3 | 2 |
| $s$ | 1 | 1 | 1 | 2 | 1 | 1 | 1 |
| $a$ | 44 | 68 | 74 | 86 | 92 | 108 | 110 |

A leapfrog fullerene when $r \equiv s \equiv 0 \pmod 3$

## $D_{3h}$ | 12 | Nanotube parameter: $s$ | $\mathcal{K}_1$





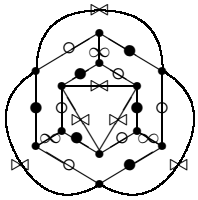

Coordinate codes:
○ $(p+r,p)$
● $(p,p+r)$
∗ $(r)$
★ $(s)$

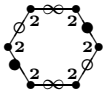
Atoms:
$a = 18p^2 + 14r^2 + 36pr + 12s(p+r)$

| $p$ | 1 | 1 | 1 | 1 | 1 | 1 | 2 |
|---|---|---|---|---|---|---|---|
| $r$ | 1 | 1 | 1 | 1 | 2 | 1 | 1 |
| $s$ | 1 | 2 | 3 | 4 | 1 | 5 | 1 |
| $a$ | 92 | 116 | 140 | 164 | 182 | 188 | 194 |

A leapfrog fullerene when $r \equiv s \equiv 0 \pmod 3$

## $D_{3h}$ | 12 | Nanotube parameter: $q$ | $\mathcal{K}_2$





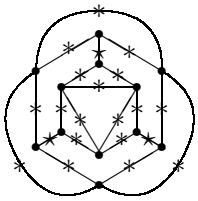

Coordinate codes:
○ $(p+r,p)$
● $(p,p+r)$
⋈ $(p,p)$
∞ $(q,q)$

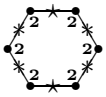
Atoms:
$a = 42p^2 + 6r^2 + 36pr + 12q(3p+r)$

| $p$ | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
|---|---|---|---|---|---|---|---|
| $q$ | 1 | 2 | 1 | 3 | 2 | 4 | 1 |
| $r$ | 1 | 1 | 2 | 1 | 2 | 1 | 3 |
| $a$ | 132 | 180 | 198 | 228 | 258 | 276 | 276 |

A leapfrog fullerene when $r \equiv 0 \pmod 3$

## $D_{3h}$ | 12 | Nanotube parameter: $s$ | $\mathcal{K}_3$





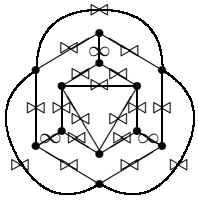

Coordinate codes:
∗ $(r)$
★ $(s)$

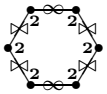
Atoms:
$a = 14r^2 + 12sr$

| $r$ | 1 | 1 | 1 | 1 | 1 | 2 | 1 |
|---|---|---|---|---|---|---|---|
| $s$ | 1 | 2 | 3 | 4 | 5 | 1 | 6 |
| $a$ | 26 | 38 | 50 | 62 | 74 | 80 | 86 |

A leapfrog fullerene when $r \equiv s \equiv 0 \pmod 3$

## $D_{3h}$ | 12 | Nanotube parameter: $q$ | $\mathcal{K}_4$





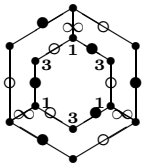

Coordinate codes:
⋈ $(p,p)$
∞ $(q,q)$

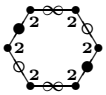
Atoms:
$a = 42p^2 + 36pq$

| $p$ | 1 | 1 | 1 | 1 | 1 | 2 | 1 |
|---|---|---|---|---|---|---|---|
| $q$ | 1 | 2 | 3 | 4 | 5 | 1 | 6 |
| $a$ | 78 | 114 | 150 | 186 | 222 | 240 | 258 |

Always a leapfrog fullerene

## $D_{3h}$ | 12 | Nanotube parameter: $q$ | $\mathcal{L}_1$



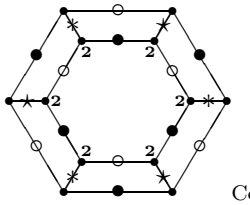

Coordinate codes:
○ $(p+r,p)$
● $(p,p+r)$
∞ $(q,q)$

Atoms:
$a = 42p^2 + 12r^2 + 48pr + 12q(3p+2r)$

| $p$ | 1 | 1 | 1 | 1 | 1 | 1 | 2 |
|---|---|---|---|---|---|---|---|
| $q$ | 1 | 2 | 1 | 3 | 4 | 2 | 1 |
| $r$ | 1 | 1 | 2 | 1 | 1 | 2 | 1 |
| $a$ | 162 | 222 | 270 | 282 | 342 | 354 | 372 |

A leapfrog fullerene when $r \equiv 0 \pmod 3$

## $D_{3h}$ | 12 | Nanotube parameter: $s$ | $\mathcal{E}_8$





Segments labeled ∗ won't belong if $s \geq 2p$.

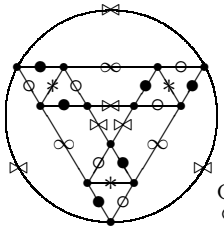
Coordinate codes:
○ $(p+r,p)$
● $(p,p+r)$
∗ $(r+s)$
★ $(s)$

Atoms:
$a = 36p^2 + 14r^2 + 48pr + 12s(2p+r)$

| $p$ | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
|---|---|---|---|---|---|---|---|
| $r$ | 1 | 1 | 1 | 2 | 1 | 1 | 2 |
| $s$ | 1 | 2 | 3 | 1 | 4 | 5 | 2 |
| $a$ | 134 | 170 | 206 | 236 | 242 | 278 | 284 |

A leapfrog fullerene when $r \equiv s \equiv 0 \pmod 3$

## $D_{3h}$ 12 $\mathcal{M}_1$




Coordinate codes:
○ $(p+r,p)$
● $(p,p+r)$
∞ $(p+s,p+s)$
⋈ $(s,s)$
∗ $(r)$

Segments labeled ∞ won't belong if $r<2s$.

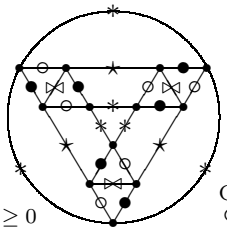
Atoms:

$a = 18p^2 + 6r^2 + 6s^2 + 24pr + 36ps + 24rs$

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| $p$ | 1 | 1 | 1 | 2 | 1 | 1 | 1 |
| $r$ | 1 | 2 | 1 | 1 | 3 | 1 | 2 |
| $s$ | 1 | 1 | 2 | 1 | 1 | 3 | 2 |
| $a$ | 114 | 180 | 192 | 228 | 258 | 282 | 282 |

A leapfrog fullerene when $r \equiv 0 (\text{mod } 3)$

## $D_{3h}$ 12 $\mathcal{M}_2$




$r \geq 0$

Coordinate codes:
○ $(p+r,p)$
● $(p,p+r)$
⋈ $(p,p)$
★ $(r+s)$
∗ $(s)$

Segments labeled ⋆ won't belong if $2p \leq s$.

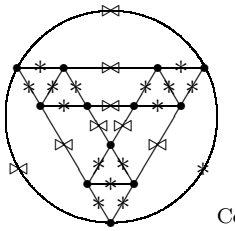
Atoms:

$a = 18p^2 + 6r^2 + 2s^2 + 24pr + 24ps + 12rs$

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| $p$ | 1 | 1 | 1 | 1 | 2 | 1 | 1 |
| $r$ | 0 | 0 | 1 | 0 | 0 | 1 | 2 |
| $s$ | 1 | 2 | 1 | 3 | 1 | 2 | 1 |
| $a$ | 44 | 74 | 86 | 108 | 122 | 128 | 140 |

A leapfrog fullerene when $r \equiv s \equiv 0 (\text{mod } 3)$

## $D_{3h}$ 12 $\mathcal{M}_3$




Coordinate codes:
⋈ $(p,p)$
∗ $(s)$

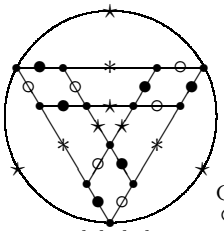
Atoms:

$a = 6p^2 + 6r^2 6 + 24pr$

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| $p$ | 1 | 1 | 2 | 1 | 3 | 2 | 1 |
| $r$ | 1 | 2 | 1 | 3 | 1 | 2 | 4 |
| $a$ | 36 | 78 | 78 | 132 | 132 | 144 | 198 |

A leapfrog fullerene when $r \equiv s \equiv 0 (\text{mod } 3)$

## $D_{3h}$ 12 $\mathcal{N}_1$




Coordinate codes:
○ $(p+r,p)$
● $(p,p+r)$
∗ $(s)$
★ $(s+r)$

Segments labeled ⋆ won't belong if $2p \leq s$.

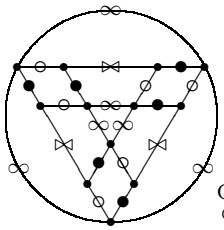
Atoms:

$a = 12p^2 + 14r^2 + 2s^2 + 36pr + 24ps + 16rs$

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| $p$ | 1 | 1 | 1 | 2 | 1 | 1 | 1 |
| $r$ | 1 | 1 | 2 | 1 | 1 | 1 | 2 |
| $s$ | 1 | 2 | 1 | 1 | 3 | 4 | 2 |
| $a$ | 104 | 150 | 198 | 200 | 200 | 254 | 260 |

A leapfrog fullerene when $r \equiv s \equiv 0 (\text{mod } 3)$

## $D_{3h}$ 12 $\mathcal{N}_2$




Coordinate codes:
○ $(p+r,p)$
● $(p,p+r)$
∞ $(p+s,p+s)$
⋈ $(s,s)$

Segments labeled ∞ won't belong if $r<2s$.

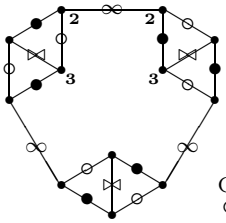
Atoms:

$a = 42p^2 + 6r^2 + 6s^2 + 36pr + 48ps + 24rs$

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| $p$ | 1 | 1 | 1 | 1 | 1 | 1 | 2 |
| $r$ | 1 | 2 | 1 | 3 | 2 | 1 | 1 |
| $s$ | 1 | 1 | 2 | 1 | 2 | 3 | 1 |
| $a$ | 162 | 240 | 252 | 330 | 354 | 354 | 372 |

A leapfrog fullerene when $r \equiv 0 (\text{mod } 3)$

## $D_{3h}$ 12 $\mathcal{O}_1$




$r \geq 0$

Coordinate codes:
○ $(p+r,p)$
● $(p,p+r)$
⋈ $(p,p)$
∞ $(s,s)$

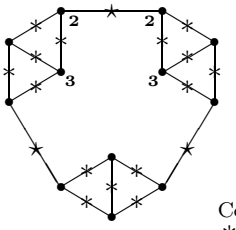
Atoms:

$a = 42p^2 + 12r^2 + 6s^2 + 60pr + 48ps + 12rs$

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| $p$ | 1 | 1 | 1 | 1 | 1 | 2 | 1 |
| $r$ | 0 | 0 | 1 | 0 | 1 | 0 | 2 |
| $s$ | 1 | 2 | 1 | 3 | 2 | 1 | 1 |
| $a$ | 96 | 162 | 180 | 240 | 258 | 270 | 288 |

A leapfrog fullerene when $r \equiv 0 (\text{mod } 3)$

## $D_{3h}$ | 12 | $\mathcal{O}_2$



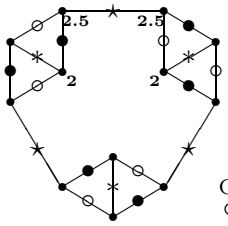

Coordinate codes:
$*$ $(r)$
$\star$ $(s)$

Atoms:
$a = 14r^2 + 2s^2 + 16rs$

| $r$ | 1 | 1 | 1 | 2 | 1 | 2 | 1 |
|---|---|---|---|---|---|---|---|
| $s$ | 1 | 2 | 3 | 1 | 4 | 2 | 5 |
| $a$ | 32 | 54 | 80 | 90 | 110 | 128 | 144 |

A leapfrog fullerene when $r \equiv 0 (\text{mod } 3)$

## $D_{3h}$ | 12 | $\mathcal{O}_3$



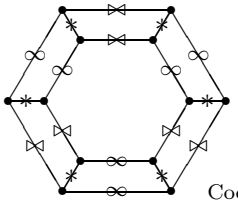

Coordinate codes:
$\circ$ $(p+r, p)$
$\bullet$ $(p, p+r)$
$*$ $(r)$
$\star$ $(s)$

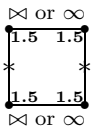
Atoms:
$a = 36p^2 + 14r^2 + 2s^2 + 48pr + 24ps + 16rs$

| $p$ | 1 | 1 | 1 | 1 | 1 | 1 | 2 |
|---|---|---|---|---|---|---|---|
| $r$ | 1 | 1 | 1 | 2 | 1 | 2 | 1 |
| $s$ | 1 | 2 | 3 | 1 | 4 | 2 | 1 |
| $a$ | 140 | 186 | 236 | 246 | 290 | 308 | 320 |

A leapfrog fullerene when $r \equiv s \equiv 0 (\text{mod } 3)$

## $D_{3h}$ | 12 | Nanotube parameter: $s$ | $\mathcal{E}_9$





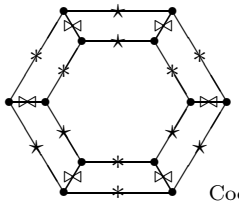

Coordinate codes:
$\infty$ $(p+r, p+r)$
$\bowtie$ $(p, p)$
$*$ $(s)$

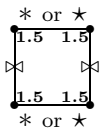
Atoms:
$a = 36p^2 + 6r^2 + 36pr + 12s(2p + r)$

| $p$ | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
|---|---|---|---|---|---|---|---|
| $r$ | 1 | 1 | 2 | 1 | 1 | 2 | 1 |
| $s$ | 1 | 2 | 1 | 3 | 4 | 2 | 5 |
| $a$ | 114 | 150 | 180 | 186 | 222 | 228 | 258 |

A leapfrog fullerene when $s \equiv 0 (\text{mod } 3)$

## $D_{3h}$ | 12 | Nanotube parameter: $p$ | $\mathcal{E}_{10}$





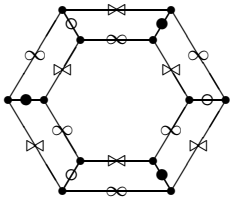

Coordinate codes:
$\bowtie$ $(p, p)$
$\star$ $(r+s)$
$*$ $(r)$

Atoms:
$a = 12r^2 + 2s^2 + 12rs + 12p(2r + s)$

| $p$ | 1 | 1 | 2 | 1 | 1 | 3 | 2 |
|---|---|---|---|---|---|---|---|
| $r$ | 1 | 1 | 1 | 1 | 2 | 1 | 1 |
| $s$ | 1 | 2 | 1 | 3 | 1 | 1 | 2 |
| $a$ | 62 | 92 | 98 | 126 | 134 | 134 | 140 |

A leapfrog fullerene when $r \equiv s \equiv 0 (\text{mod } 3)$

## $D_{3d}$ | 12 | Nanotube parameter: $q$ | $\mathcal{E}_{11}$




Coordinate codes:
$\circ$ $(q, r)$
$\bullet$ $(r, q)$
$\bowtie$ $(p, p)$
$\infty$ $(p+r, p+r)$

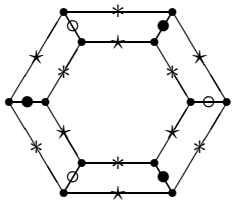
Segments labeled $\infty$ won't belong if $q < 2p + r$.

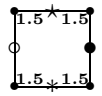
Atoms:
$a = 36p^2 + 12r^2 + 48pr + 12q(2p + r)$

| $p$ | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
|---|---|---|---|---|---|---|---|
| $q$ | 1 | 2 | 3 | 1 | 4 | 2 | 5 |
| $r$ | 1 | 1 | 1 | 2 | 1 | 2 | 1 |
| $a$ | 132 | 168 | 204 | 228 | 240 | 276 | 276 |

A leapfrog fullerene when $q \equiv r (\text{mod } 3)$

## $D_{3d}$ | 12 | Nanotube parameter: $p$ | $\mathcal{E}_{12}$






Coordinate codes:
$\circ$ $(p+r, p)$
$\bullet$ $(p, p+r)$
$\star$ $(r+s)$
$*$ $(s)$

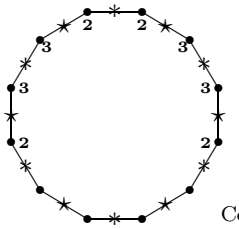
Segments labeled $\star$ won't belong if $2p \leq s$.

Atoms:
$a = 8r^2 + 12s^2 + 24rs + 12p(r + 2s)$

| $p$ | 1 | 2 | 1 | 3 | 1 | 2 | 4 |
|---|---|---|---|---|---|---|---|
| $r$ | 1 | 1 | 2 | 1 | 1 | 2 | 1 |
| $s$ | 1 | 1 | 1 | 1 | 2 | 1 | 1 |
| $a$ | 80 | 116 | 140 | 152 | 164 | 188 | 188 |

A leapfrog fullerene when $r \equiv s \equiv 0 (\text{mod } 3)$

## $D_{3d}$ | 12 | $\mathcal{D}_{14}$



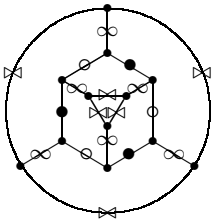

Coordinate codes:
$*$ $(s)$
$\star$ $(r)$

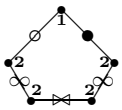
Atoms:
$a = 8r^2 + 12s^2 + 24rs$

| $r$ | 1 | 2 | 1 | 3 | 2 | 1 | 4 |
|---|---|---|---|---|---|---|---|
| $s$ | 1 | 1 | 2 | 1 | 2 | 3 | 1 |
| $a$ | 44 | 92 | 104 | 156 | 176 | 188 | 236 |

A leapfrog fullerene when $r \equiv s \equiv 0 (\text{mod } 3)$

## $D_{3d}$ | 12 | Nanotube parameter: $p$ | $\mathcal{P}_1$






Coordinate codes:
$\circ$ $(r+s, p)$
$\bullet$ $(p, r+s)$
$\bowtie$ $(r, r)$
$\infty$ $(s, s)$

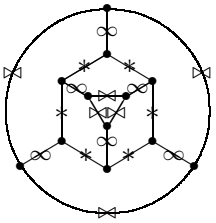
Segments labeled $\bowtie$ won't belong if $r > p + s$.

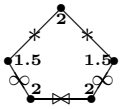
Atoms:
$a = 12r^2 + 24s^2 + 48rs + 12p(r+s)$

| $p$ | 1 | 2 | 3 | 4 | 5 | 1 | 6 |
|---|---|---|---|---|---|---|---|
| $r$ | 1 | 1 | 1 | 1 | 1 | 2 | 1 |
| $s$ | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| $a$ | 108 | 132 | 156 | 180 | 204 | 204 | 228 |

A leapfrog fullerene when $(r+s) \equiv p (\text{mod } 3)$

## $D_{3d}$ | 12 | $\mathcal{P}_2$






Coordinate codes:
$*$ $(r+s)$
$\bowtie$ $(r, r)$
$\infty$ $(s, s)$

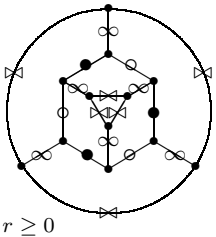
Segments labeled $\bowtie$ won't belong if $r > s$.

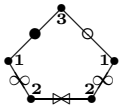
Atoms:
$a = 12r^2 + 24s^2 + 48rs$

| $r$ | 1 | 2 | 1 | 3 | 2 | 1 | 4 |
|---|---|---|---|---|---|---|---|
| $s$ | 1 | 1 | 2 | 1 | 2 | 3 | 1 |
| $a$ | 84 | 168 | 204 | 276 | 336 | 372 | 408 |

A leapfrog fullerene when $(r+s) \equiv 0 (\text{mod } 3)$

## $D_{3d}$ | 12 | $\mathcal{P}_3$






Coordinate codes:
$\circ$ $(p+r+s, p)$
$\bullet$ $(p, r+s+p)$
$\bowtie$ $(p+r, p+r)$
$\infty$ $(p+s, p+s)$

$r \geq 0$

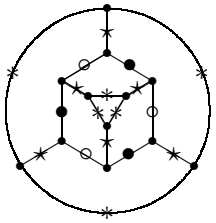
Segments labeled $\bowtie$ won't belong if $r > s$.

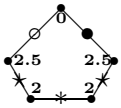
Atoms:
$a = 60p^2 + 12r^2 + 24s^2 + 60pr + 84ps + 48rs$

| $p$ | 1 | 1 | 1 | 1 | 2 | 1 | 1 |
|---|---|---|---|---|---|---|---|
| $r$ | 0 | 1 | 0 | 2 | 0 | 1 | 0 |
| $s$ | 1 | 1 | 2 | 1 | 1 | 2 | 3 |
| $a$ | 168 | 228 | 324 | 432 | 432 | 528 | 600 |

A leapfrog fullerene when $(r+s) \equiv p (\text{mod } 3)$

## $D_{3d}$ | 12 | Nanotube parameter: $p$ | $\mathcal{P}_4$





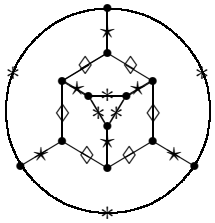

Coordinate codes:
$\circ$ $(p+r+s, p)$
$\bullet$ $(p, p+r+s)$
$*$ $(r)$
$\star$ $(s)$

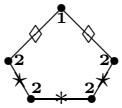
Atoms:
$a = 8r^2 + 12s^2 + 24rs + 12p(r+s)$

| $p$ | 1 | 2 | 3 | 1 | 1 | 4 | 2 |
|---|---|---|---|---|---|---|---|
| $r$ | 1 | 1 | 1 | 2 | 1 | 1 | 2 |
| $s$ | 1 | 1 | 1 | 1 | 2 | 1 | 1 |
| $a$ | 68 | 92 | 116 | 128 | 140 | 140 | 164 |

A leapfrog fullerene when $r \equiv s \equiv 0 (\text{mod } 3)$

## $D_{3d}$ | 12 | $\mathcal{P}_5$





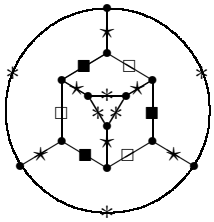

Coordinate codes:
$\Diamond$ $(r+s)$
$*$ $(r)$
$\star$ $(s)$

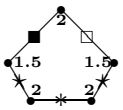
Atoms:
$a = 8r^2 + 12s^2 + 24rs$

| $r$ | 1 | 2 | 1 | 3 | 2 | 1 | 4 |
|---|---|---|---|---|---|---|---|
| $s$ | 1 | 1 | 2 | 1 | 2 | 3 | 1 |
| $a$ | 44 | 92 | 104 | 156 | 176 | 188 | 236 |

A leapfrog fullerene when $r \equiv s \equiv 0 (\text{mod } 3)$

## $D_{3d}$ 12 $\mathcal{P}_6$






Coordinate codes:
□ $(r+s,p)$
■ $(p,r+s)$
∗ $(r)$
★ $(2p+s)$

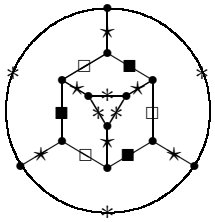
$s \geq 0$

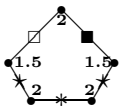
Atoms:
$a = 24p^2 + 8r^2 + 12s^2 + 36pr + 36ps + 24rs$

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| $p$ | 1 | 1 | 1 | 2 | 1 | 1 | 1 |
| $r$ | 1 | 2 | 1 | 1 | 3 | 2 | 1 |
| $s$ | 0 | 0 | 1 | 0 | 0 | 1 | 2 |
| $a$ | 68 | 128 | 140 | 176 | 204 | 224 | 236 |

A leapfrog fullerene when $p \equiv s$ & $r \equiv 0$(mod 3)

## $D_{3d}$ 12 $\mathcal{P}_7$






Coordinate codes:
□ $(r+s,p)$
■ $(p,r+s)$
∗ $(p+s)$
★ $(2r+s)$

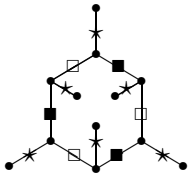
$r \geq 0$
Segments labeled ∗ won't belong if $r = 0$.

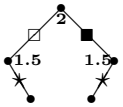
Atoms:
$a = 8p^2 + 24r^2 + 20s^2 + 36pr + 28ps + 48rs$

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| $p$ | 1 | 2 | 1 | 1 | 3 | 2 | 2 |
| $r$ | 0 | 0 | 0 | 1 | 0 | 0 | 1 |
| $s$ | 1 | 1 | 2 | 1 | 1 | 2 | 1 |
| $a$ | 56 | 108 | 144 | 164 | 176 | 224 | 252 |

A leapfrog fullerene when $r \equiv s \equiv -p$(mod 3)

## $D_{3d}$ 12 $\mathcal{Q}_1$





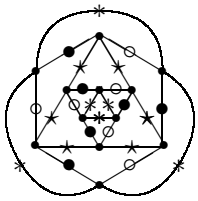

Coordinate codes:
□ $(r+s,p)$
■ $(p,r+s)$
★ $(s)$

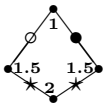
Atoms:
$a = 8p^2 + 20r^2 + 8s^2 + 28pr + 20ps + 32rs$

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| $p$ | 1 | 2 | 1 | 1 | 3 | 2 | 1 |
| $r$ | 1 | 1 | 1 | 2 | 1 | 1 | 1 |
| $s$ | 1 | 1 | 2 | 1 | 1 | 2 | 3 |
| $a$ | 116 | 188 | 192 | 236 | 276 | 284 | 284 |

A leapfrog fullerene when $p \equiv r$ & $s \equiv 0$(mod 3)

## $D_{3d}$ 12 Nanotube parameter: $q$ $\mathcal{R}_1$





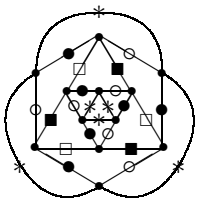

Coordinate codes:
○ $(p+r,p)$
● $(p,p+r)$
★ $(p+r)$
∗ $(r)$

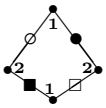
Atoms:
$a = 24p^2 + 20r^2 + 48pr$

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| $p$ | 1 | 1 | 2 | 1 | 2 | 3 | 1 |
| $r$ | 1 | 2 | 1 | 3 | 2 | 1 | 4 |
| $a$ | 92 | 200 | 212 | 348 | 368 | 380 | 536 |

A leapfrog fullerene when $p \equiv r \equiv 0$(mod 3)

## $D_{3d}$ 12 Nanotube parameter: $q$ $\mathcal{R}_2$





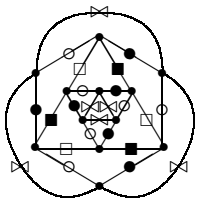

Coordinate codes:
○ $(p+r,p)$
● $(p,p+r)$ ∗ $(r)$
□ $(p+r,q)$
■ $(q,p+r)$

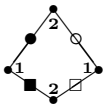
Atoms:
$a = 24p^2 + 20r^2 + 48pr + 12q(p+r)$

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| $p$ | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| $q$ | 1 | 2 | 3 | 4 | 5 | 6 | 1 |
| $r$ | 1 | 1 | 1 | 1 | 1 | 1 | 2 |
| $a$ | 116 | 140 | 164 | 188 | 212 | 236 | 236 |

A leapfrog fullerene when $p \equiv q$ & $r \equiv 0$(mod 3) and (mod 3)

## $D_{3d}$ 12 $\mathcal{R}_3$






Coordinate codes:
○ $(p+2r+s,p)$
● $(p,p+2r+s)$
⋈ $(p,p)$
□ $(p+r,p+s)$
■ $(p+s,p+r)$

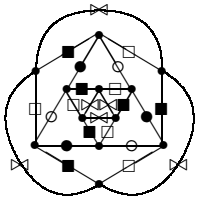
$r \geq 0$

Atoms:
$a = 60p^2 + 24r^2 + 12s^2 + 84pr + 60ps + 36rs$

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| $p$ | 1 | 1 | 1 | 1 | 2 | 1 | 1 |
| $r$ | 0 | 0 | 1 | 0 | 0 | 1 | 2 |
| $s$ | 1 | 2 | 1 | 3 | 1 | 2 | 1 |
| $a$ | 132 | 228 | 276 | 348 | 372 | 408 | 468 |

A leapfrog fullerene when $r \equiv s$(mod 3)

$D_{3d}$ | 12 | | $\mathcal{R}_4$




Coordinate codes:
○ $(p+2r+s, p)$
● $(p, p+2r+s)$
⋈ $(p, p)$
□ $(p+r, p+s)$
■ $(p+s, p+r)$

Segments labeled ⋈ won't belong if $r > s$.

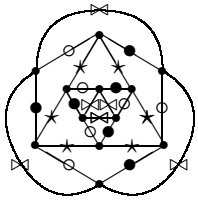
Atoms:

$a = 60p^2 + 36r^2 + 12s^2 + 96pr + 60ps + 48rs$

| $p$ | 1 | 1 | 1 | 1 | 2 | 1 | 1 |
|---|---|---|---|---|---|---|---|
| $r$ | 1 | 1 | 2 | 1 | 1 | 2 | 1 |
| $s$ | 1 | 2 | 1 | 3 | 1 | 2 | 4 |
| $a$ | 312 | 456 | 564 | 624 | 648 | 756 | 816 |

A leapfrog fullerene when $r \equiv s \pmod 3$

$D_{3d}$ | 12 | | $\mathcal{R}_5$




Coordinate codes:
○ $(r, p)$
● $(p, r)$
★ $(2p+r)$
⋈ $(p, p)$

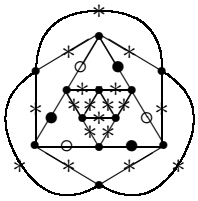
Atoms:

$a = 36p^2 + 12r^2 + 72pr$

| $p$ | 1 | 1 | 2 | 1 | 2 | 1 | 3 |
|---|---|---|---|---|---|---|---|
| $r$ | 1 | 2 | 1 | 3 | 2 | 4 | 1 |
| $a$ | 96 | 180 | 252 | 288 | 384 | 420 | 480 |

A leapfrog fullerene when $r \equiv 0 \pmod 3$

$D_{3d}$ | 12 | Nanotube parameter: $q$ | $\mathcal{R}_6$




Coordinate codes:
○ $(p, q)$
● $(q, p)$
∗ $(p)$

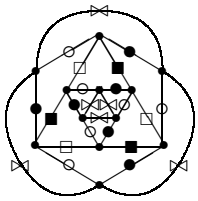
Atoms:

$a = 20p^2 + 12pq$

| $p$ | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
|---|---|---|---|---|---|---|---|
| $q$ | 1 | 2 | 3 | 4 | 5 | 6 | 7 |
| $a$ | 32 | 44 | 56 | 68 | 80 | 92 | 104 |

A leapfrog fullerene when $p \equiv q \pmod 3$

$D_{3d}$ | 12 | Nanotube parameter: $q$ | $\mathcal{R}_7$




Coordinate codes:
○ $(r, p)$
● $(p, r)$
⋈ $(p, p)$
□ $(q+2p+r, q)$
■ $(q, q+2p+r)$

Segments labeled ⋈ won't belong if $p > r$.

Atoms:

$a = 36p^2 + 12r^2 + 48pr + 12q(2p+r)$

| $p$ | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
|---|---|---|---|---|---|---|---|
| $q$ | 1 | 2 | 3 | 1 | 4 | 5 | 2 |
| $r$ | 1 | 1 | 1 | 2 | 1 | 1 | 2 |
| $a$ | 132 | 168 | 204 | 228 | 240 | 276 | 276 |

A leapfrog fullerene when $p \equiv r \pmod 3$

## References


[1] D. L. D. Caspar and A. Klug, *Viruses, nucleic acids and cancer*, 17th Anderson Symposium, Williams & Wilkins, Baltimore, 1963.

[2] H. S. M Coxeter, Virus macromolecules and geodesic domes, in (J. C. Butcher, ed.), *A Spectrum of Mathematics*, Oxford Univ. Press, 1971, 98–107.

[3] P. W. Fowler, J. E. Cremona, and J. I. Steer, Systematics of bonding in non-icosahedral carbon clusters, *Theor. Chim. Acta* **73** (1988), 1–26.

[4] P. W. Fowler, and D. E. Manolopoulos, *An Atlas of Fullerenes*, Clarenden Press, Oxford, 1995.

[5] Michael Goldberg, A class of multi-symmetric polyhedra, *Tohoku Math. J.* **43** (1939), 104–108.

[6] J. E. Graver, Encoding Fullerenes and Geodesic Domes, *SIAM J. Discrete Math.* **17**(4) (2004), 596–614.

[7] J. E. Graver, The structure of fullerene signatures, this volume.



Department of Mathematics, Syracuse University, Syracuse, NY 13244-1150
*E-mail address*: jegraver@syr.edu

# CORRECTIONS TO A CATALOG OF ALL FULLERENES WITH TEN OR MORE SYMMETRIES

Since the publication of this paper, researchers who have use the catalog and have found and corrected several errors. Most of these corrections are due to Giuseppe Mazzuoccolo and Mathieu Bogaerts. I am very appreciative of their careful reading and would be glad to hear of any other errors that are found. JEG

**page 2, Figure 1:**
The three segments labeled (3,4) should be labeled (2,3).

**page 5, Figure $\mathcal{A}_{11}$:**
The order of the group is 12

**page 7, Figure $\mathcal{C}_{10}$:**
A leapfrog fullerene when $r \equiv s \equiv 0 \pmod 3$

**page 7, Figure $\mathcal{B}_3$:**
A leapfrog fullerene when $q + s \equiv 0$ & $r \equiv s \pmod 3$

**page 7, Figure $\mathcal{B}_4$:**
A leapfrog fullerene when $r \equiv s \equiv 0 \pmod 3$

**page 18, Figure $\mathcal{M}_3$:**
The parameter for $*$ should be $(r)$ not $(s)$.
$a = 6p^2 + 6r^2 + 24pr$
A leapfrog fullerene when $r \equiv 0 \pmod 3$

**page 19, Figure $\mathcal{O}_2$:**
A leapfrog fullerene when $r \equiv s \equiv 0 \pmod 3$

**page 21, Figure $\mathcal{Q}_1$:**
The parameter for $\star$ should be $(r)$ not $(s)$.
A leapfrog fullerene when $p \equiv s \pmod 3$ and $r \equiv 0 \pmod 3$

**page 22, Figure $\mathcal{R}_4$:**
The parameter for $\bowtie$ should be $(p + r, p + r)$ not $(p, p)$.

**page 22, Figure $\mathcal{R}_6$:**
A leapfrog fullerene when $p \equiv q \equiv 0 \pmod 3$