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54

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Combinatorics with Emphasis on the Theory of Graphs

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To our Fathers,

Harold John Graver

Misha Mark Watkins (in memory)

Preface

Combinatorics and graph theory have mushroomed in recent years. Many overlapping or equivalent results have been produced. Some of these are special cases of unformulated or unrecognized general theorems. The body of knowledge has now reached a stage where approaches toward unification are overdue. To paraphrase Professor Gian-Carlo Rota (Toronto, 1967), "Combinatorics needs fewer theorems and more theory."

In this book we are doing two things at the same time:

A. We are presenting a unified treatment of much of combinatorics and graph theory. We have constructed a concise algebraically-based, but otherwise self-contained theory, which at one time embraces the basic theorems that one normally wishes to prove while giving a common terminology and framework for the development of further more specialized results.

B. We are writing a textbook whereby a student of mathematics or a mathematician with another specialty can learn combinatorics and graph theory. We want this learning to be done in a much more unified way than has generally been possible from the existing literature.

Our most difficult problem in the course of writing this book has been to keep A and B in balance. On the one hand, this book would be useless as a textbook if certain intuitively appealing, classical combinatorial results were either overlooked or were treated only at a level of abstraction rendering them beyond all recognition. On the other hand, we maintain our position that such results can all find a home as part of a larger, more general structure.

To convey more explicitly what this text is accomplishing, let us compare combinatorics with another mathematical area which, like combinatorics, has been realized as a field in the present century, namely topology. The basic unification of topology occurred with the acceptance of what we now call a "topology" as the underlying object. This concept was general enough to encompass most of the objects which people wished to study, strong enough to include many of the basic theorems, and simple enough so that additional conditions could be added without undue complications or repetition.

We believe that in this sense the concept of a "system" is the right unifying choice for combinatorics and graph theory. A system consists of a finite set of objects called "vertices," another finite set of objects called "blocks," and an "incidence" function assigning to each block a subset of the set of vertices. Thus graphs are systems with blocksize two; designs are systems with constant blocksize satisfying certain conditions; matroids are also systems; and a system is the natural setting for matchings and inclusion-exclusion. Some important notions are studied in this most general setting, such as connectivity and orthogonality as well as the parameters and vector spaces of a system. Connectivity is important in both graph theory and matroid theory, and parallel theorems are now avoided. The vector spaces of a system have important applications in all of these topics, and again much duplication is avoided.

One other unifying technique employed is a single notation consistent throughout the book. In attempting to construct such a notation, one must face many different levels in the hierarchy of sets (elements, sets of elements, collections of sets, families of collections, etc.) as well as other objects (systems, functions, sets of functions, lists, etc.). We decided insofar as possible to use different types of letters for different types of objects. Since each topic covered usually involves only a few types of objects, there is a strong temptation to adopt a simpler notation for that section regardless of how it fits in with the rest of the book. We have resisted this temptation. Consequently, once the notational system is mastered, the reader will be able to flip from chapter to chapter, understanding at glance the diverse roles played in the middle and later chapters by the concepts introduced in the earlier chapters.

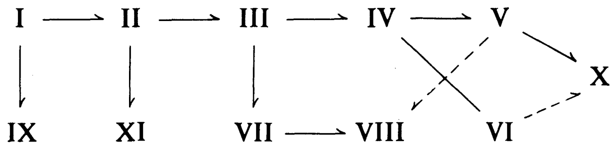
An undergraduate course in linear algebra is prerequisite to the comprehension of most of this book. Basic group theory is needed for sections *II*E and *XI*C. A deeper appreciation of sections *III*E, *III*G, *VII*C, and *VII*D will be gained by the reader who has had a year of topology. All of these sections may be omitted, however, without destroying the continuity of the rest of the text.

The level of exposition is set for the beginning graduate student in the mathematical sciences. It is also appropriate for the specialist in another mathematical field who wishes to learn combinatorics from scratch but from a sophisticated point of view.

It has been our experience while teaching from the notes that have evolved into this text, that it would take approximately three semesters of three hours classroom contact per week to cover all of the material that we have presented. A perusal of the Table of Contents and of the "Flow Chart of the

Sections" following this Preface will suggest the numerous ways in which a subset of the sections can be covered in a subset of three semesters. A List of Symbols and an Index of Terms are provided to assist the reader who may have skipped over the section in which a symbol or term was defined.

As indicated in the figure below, a one-semester course can be formed from Chapters I, II, IX, and XI. However, the instructor must provide some elementary graph theory in a few instances. The dashed lines in the figure below as well as in the Flow Chart of the Sections indicate a rather weak dependency.



If a two-semester sequence is desired, we urge that Chapters I, II, and III be treated in sequence in the first semester, since they comprise the theoretical core of the book. The reader should not be discouraged by the apparent dryness of Chapter II. There is a dividend which is compounded and paid back chapter by chapter. We recommend also that Chapters IV, V, and VI be studied in sequence; they are variations on a theme, a kind of minimax or maximin principle, which is an important combinatorial notion. Since Chapter X brings together notions from the first six chapters with allusions to Chapters VII and IX, it would be a suitable finale.

There has been no attempt on our part to be encyclopedic. We have even slighted topics dear to our respective hearts, such as integer programming and automorphism groups of graphs. We apologize to our colleagues whose favorite topics have been similarly slighted.

There has been a concerted effort to keep the technical vocabulary lean. Formal definitions are not allotted to terms which are used for only a little while and then never again. Such terms are often written between quotation marks. Quotation marks are also used in intuitive discussions for terms which have yet to be defined precisely.

The terms which do form part of our technical vocabulary appear in **bold-face** type when they are formally defined, and they are listed in the Index.

There are two kinds of exercises. When the term "**Exercise**" appears in bold-face type, then those assertions in italics following it will be invoked in subsequent arguments in the text. They almost always consist of straightforward proofs with which we prefer not to get bogged down and thereby lose too much momentum. The word "*Exercise*" (in italics) generally indicates a specific application of a principle, or it may represent a digression which the limitations of time and space have forced us not to pursue. In principle, all of the exercises are important for a deeper understanding of and insight into the theory.

Chapters are numbered with Roman numerals; the sections within each chapter are denoted by capital letters; and items (theorems, exercises, figures, etc.) are numbered consecutively regardless of type within each section. If an item has more than one part, then the parts are denoted by lower case Latin letters. For references within a chapter, the chapter number will be suppressed, while in references to items in other chapters, the chapter number will be italicized. For example, within Chapter III, Euler's Formula is referred to as F2b, but when it is invoked in Chapter VII, it is denoted by *III*F2b.

Relatively few of the results in this text are entirely new, although many represent new formulations or syntheses of published results. We have also given many new proofs of old results and some new exercises without any special indication to this effect. We have done our best to give credit where it is due, except in the case of what are generally considered to be results "from the folklore".

A special acknowledgement is due our typist, Mrs. Louise Capra, and to three of our former graduate students who have given generously of their time and personal care for the well-being of this book: John Kevin Doyle, Clare Heidema, and Charles J. Leska. Thanks are also due to the students we have had in class, who have learned from and taught us from our notes. Finally, we express our gratitude to our families, who may be glad to see us again.

Syracuse, N.Y.
April, 1977

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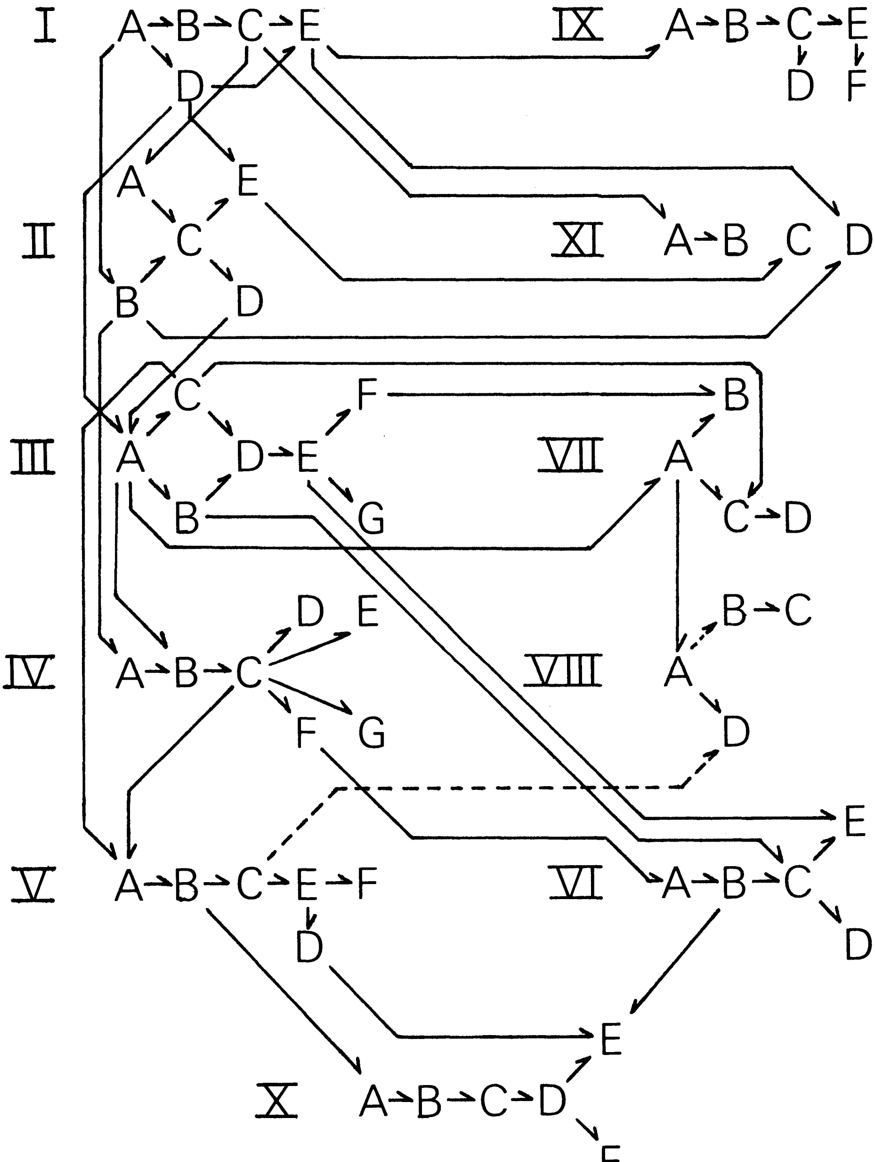

Flow Chart of the Sections

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